

# Astrophysics of Compact Objects, Fall 2011

## Final Exam

Due: 12 December 2011

Please find three contiguous hours to take the test. This exam is closed book. Books, notes, google, wikipedia, etc. are not allowed.

I'm looking in these questions for *estimates*, not detailed calculations. In many cases, I do not provide numbers (e.g., for the total energy released in AGN); that is part of the problem/question. In such cases make your assumptions clear (e.g., about the values you use for certain quantities or parameters), and then note clearly how your final result depends on those assumptions.

**Choose SIX of the eleven problems below.**

1. Derive a rough expression for the upper limit to the mass of a white dwarf in terms of fundamental constants.
2. Gamma rays with an energy  $E_\gamma \gtrsim 100$  GeV have been reportedly observed from supernova remnants in the Milky Way. What are the possible physical mechanisms for producing such photons?
3. Suppose that AGN are somehow responsible for accelerating ultra-high-energy cosmic rays (UHECRs). What fraction (*roughly*) of the total energy released in AGN goes into UHECRs.
4. In class we derived the mass-accretion rate for spherically-symmetric accretion from a gas of density  $\rho$  and sound speed  $c_s$  (far away from the accreting source) to be  $\dot{M} \simeq \pi G^2 M^2 \rho / c_s^3$ . Provide an approximate derivation of this result.
5. Stellar-population models suggest that there should be roughly  $10^8$  stellar-mass (let's say  $3 M_\odot$ ) black holes in the Galaxy. *Estimate* the flux of x-ray radiation from accretion of matter from the interstellar medium onto the nearest such black hole (the accretion rate in the previous problem may be useful). Is this within the range of detectability of any current x-ray telescope?
6. A gamma-ray burst goes off at a cosmological distance. What angular resolution would a radio telescope require to resolve the afterglow after three months?

7. Consider the internal-shock model for gamma-ray bursts: Two shells of similar mass and Lorentz factor  $\gamma_1$  and  $\gamma_2$  collide. Use conservation of energy and momentum to find the fraction of energy that will thermalize.
8. The cosmic-ray energy density in the Milky Way has an energy density comparable to that in the  $\sim \mu\text{G}$  magnetic field. Use this fact and the fact that the cosmic-ray energy spectrum is  $dN/dE \propto E^{-p}$  (with  $p \simeq 2.7$ ) to estimate the flux from the Galactic Center of neutral pions produced by collisions of cosmic-ray protons with ISM protons. Assume a density of protons in the interstellar medium of  $\text{cm}^{-3}$ .
9. In an interacting binary, the more massive star accretes matter from a less massive companion. Assume angular-momentum-conserving mass transfer. Does the distance between the stars increase or decrease? Give a simple physical explanation for why. How does the result change if angular momentum is lost from the system?
10. If a gamma ray has a sufficiently large energy, it can scatter from a cosmic-infrared-background (CIB) photon to produce an electron-positron pair. Calculate the energy  $E_\gamma$  required of a gamma ray if it is to pair produce by scattering from a CIB photon; use a wavelength of one micron for the CIB photon. Estimate *roughly* the energy density  $\rho_{\text{CIB}}$  in the CIB; make your assumptions clear. Estimate the mean-free path for gamma rays above the pair-production threshold to pair produce as they travel through the CIB. Write your answer in terms of  $\rho_{\text{CIB}}$  and evaluate it.
11. Recall that the merger of two neutron stars at a distance  $d \sim 200 \text{ Mpc}$  produces a gravitational-wave signal at the threshold  $h \sim 10^{-21}$  for detection with LIGO at  $\sim \text{kHz}$  frequencies. Estimate the mass and arm length of a barbell which, if placed 10 km from LIGO, would, when spun at kHz frequencies, would produce a detectable signal in LIGO.