Gamma-Ray Bursts: (source: notes provided by R. Sari)

Are ~~very~~ very bright ~~flash~~ (e.g., 10^{-6} erg/cm²) bursts of γ-rays (e.g., 50 keV – 10 MeV; maybe higher?) that last ~2 sec — 100s of secs (are also some with durations $0.01s \lesssim t \lesssim 2s$, but these are now thought to be something else). Light curves highly variable and often irregularly so; Variability has been observed on timescales $t \simeq$ msec. No two look alike.

Spectra:

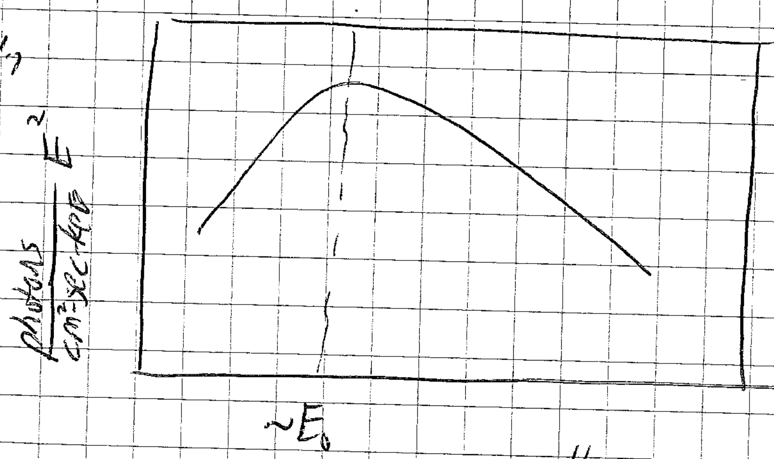
Again, are highly irregular and vary over course of the burst. Still, often parametrized in terms of 4-parameter "Band function" (Band 1993):

$$N(E) = \begin{cases} A E^{\alpha} e^{-E/E_0} & E < (\alpha-\beta)E_0 \\ A[(\alpha-\beta)E_0]^{\alpha-\beta} e^{(\beta-\alpha)} E^{\beta} & E > (\alpha-\beta)E_0 \end{cases}$$

e.g., typical values are $100\,keV \lesssim E_0 \lesssim 400\,keV$, $\alpha \simeq -1$, $\beta \simeq -2.25$

i.e., two power laws $\propto E^{\alpha}$ at low E and E^{β} at high E

e.g.,



High-energy power-law usually extends as high as can be seen; e.g. 200 MeV for GRB 930506, ~10^3x peak energy

$\Rightarrow$ highly nonthermal $\Rightarrow$ source must be optically thin!

Rate: ~ 200/yr at $f \gtrsim 10^{-7}$ erg/cm²/sec

Distribution: consistent with isotropic

$\Rightarrow$ cosmological; have redshifts $0 < z \leq 8.4$

Energies: If distance is $D \sim 3$ Gpc

$E = 4\pi D^2 F/(1+z)$ (if energy radiated isotropically)

e.g., for $F = 10^{-6}$ erg/cm², $E = 10^{51}$ erg; many as high as 10^{54} erg.

Compactness Problem:

A variability timescale Δt implies a source size $R < c\Delta t$. If the energy in γ's of $\gtrsim$ MeV is E, the optical depth, as a lower-energy γ passes through the source, to produce e^+e^- via $\gamma+\gamma \to e^+e^-$, is

$$\tau = \sigma_T \frac{N}{4\pi R^2} = 3\times 10^{12}\left(\frac{\Delta t}{0.1\,\text{sec}}\right)^{-2}\left(\frac{E}{10^{51}\,\text{erg}}\right) \gg 1$$

(Ruderman 1974).

Would therefore produce e^+e^- pairs which would then thermalize the γ's; conflicts with power-law N(E).

Relativistic Outflows:

Solve compactness problem if emitting regions move toward us at Lorentz factor $\gamma \gg 1$.

First of all, $\Delta t_{obs} = \frac{\Delta t_{em}}{2\gamma^2}$ ($\gamma \gg 1$), as in AGN jets; this gives factor $\tau \propto \gamma^{-4}$.

Moreover, ~~if observed spectrum is~~, e.g., $\frac{dN}{dE} \propto E^{-2}$, the observed energy is $E_{obs} = \gamma E$

~~Photon number~~
Therefore, number of γ's with energy above e^+e^- threshold in the emitter (moving) frame is smaller than naive estimate. This adds an additional γ^{-2} for flat-spectrum sources $(dN/E) \propto E^{-2}$.

$$\Rightarrow \quad \tau \propto \gamma^{-6} \quad \Rightarrow \quad \boxed{\gamma \gtrsim 100}$$

<u>The Fireball Model:</u>

<u>Source:</u> remains a mystery but must have (1) enough Energy, (2) correct rate of occurrence; and (3) compact. Finally, (4) the source must be "clean"; i.e., it must solve the <u>baryon pollution problem</u>:

suppose we have total energy E ~~in a small~~ (in γ's, baryons, e^+e^-) initially in small region of baryon rest mass $M = Nm_p$. The maximum achievable Lorentz factor is $\gamma_{max} = E/Mc^2$. A given Lorentz factor thus requires a baryonic mass,

$$M \leq \frac{E}{\gamma c^2} = 6 \times 10^{-6} M_\odot \left(\frac{E}{10^{51}\,\mathrm{erg}}\right)\left(\frac{100}{\gamma}\right).$$

Candidate sources include

(1) Neutron-star mergers (2) Failed supernovae (collapsar)

(3) Collapse of magnetized WD (4) Explosion of massive star - hypernova

Associations of SN1998bw $\leftarrow$ GRB 980425

and SN2003dh $\leftarrow$ GRB 030329

$\Rightarrow$ GRBs (at least some) are some type of SN; perhaps of rotating star that collapses ~~to a~~ BH + accretion disk

Fireball model: general picture:

An initially optically thick region of material expands under its own pressure until it gets $\gamma = E/M$, whereupon it becomes optically thin.

Evolution:

Assume a relativistic expand shell of material of pressure p, baryon rest-mass density ρ, width Δ (observer frame), Lorentz factor γ, and radius R.

$$\gamma^2(4p/3 + \rho c^2)\Delta R^2 = \text{const} \qquad \text{(conservation of } E\text{)}$$

$$\gamma \rho \Delta R^2 = \text{const} \qquad \text{(cons. of baryons)}$$

$$p(\gamma \Delta R^2)^{4/3} = \text{const} \qquad \text{(cons. of entropy)}$$

assuming EOS, $e = 3p$.

Solutions are an initial radiation-dominated phase with $\Delta = \text{const}$, $\gamma \propto R$, $\rho \propto R^{-3}$, followed by a matter-dominated phase with $\Delta = \text{const}$, $\gamma = \text{const}$, $\rho \propto R^{-2}$, until $R/\gamma^2 \gtrsim \Delta$, at which point shell spreads in width with $\Delta = R/\gamma^2$

Thermalizing the energy:

The relativistic shell must run into something to thermalize the ~~g~~ material and emit γ-rays.
Two possibilities:

① external shocks: the shell runs into the ISM

② internal shocks: flow is irregular and multiple ~~many~~ shells collide with each other

External shocks have trouble providing variability, but may be responsible for late-time lower-ν afterglows.

Internal shocks require the source to be variable.

Suppose the outflow is composed of many relativistic shells ~~with~~ seperated by δ, and let Δ be the distance between the first and last shell.

Suppose two adjacent shells have Lorentz factors γ and 2γ. They collide at distance $R \sim \gamma^2 \delta$ and produce pulse of duration $\Delta t = R/\gamma^2 c = \delta/c$

Converting Thermal Energy to Radiation:

The ~~co~~ shell collisions result in relativistic shocks ~~and~~ which ~~produce strong~~ accelerate e^-'s and produce $\vec{B}$ fields via plasma instabilities. The observed γ-rays are then synchrotron radiation and inverse-Compton-scattered synchrotron γ's. The typical frequency ~~from~~ from an e^- of Lorentz factor γ_e is

$$\nu = \frac{1}{2\pi} \frac{eB}{m_e c} \gamma_e^2$$

the power of each e^- is $P = \frac{4}{3} \sigma_T c \gamma_e^2 \frac{B^2}{8\pi}$

GRB Afterglows:

the initial ~~nmin~~ GRB is followed over long times ~~to~~ (~months) by radiation at lower frequencies (X-ray, UV, optical, IR, radio), with diminishing intensity and frequency.

These afterglows are believed to be due to the external shock produced by the interaction of the ~~outfl~~ relativistic outflow into the ISM; like a relativistic SNR.

Suppose the initial ISM has uniform proton density n and that the shock moves through it. ~~at~~ Behind the shock, the particle density is $4\gamma n$ and the energy density is $4\gamma^2 n m_p c^2$, where γ is the Lorentz factor of the shocked fluid.

We assume that Fermi acceleration produces a power-law distribution of e^- Lorentz factors γ_e (in the fluid rest frame) of

$$N(\gamma_e)\, d\gamma_e \propto \gamma_e^{-p}\, d\gamma_e \quad \text{for } \gamma_e \geq \gamma_m,$$

and we take ~~p≥2~~ $p > 2$. Assume a constant fraction ϵ_e of the shock energy goes into e^-'s. Then

$$\gamma_m = \epsilon_e \left(\frac{p-2}{p-1}\right) \frac{m_p}{m_e} \gamma \simeq 610\, \epsilon_e \gamma. \quad \text{for } p = 2.5$$

We also assume that a fraction ϵ_B of the shock energy goes into the B field:

$$B = (32\pi m_p \epsilon_B n)^{1/2} \gamma c.$$

Synchrotron Spectrum of Relativistic Shock:

~~If~~ the observed power and characteristic frequency are from an e^- of ~~...~~ rest-frame γ_e moving with bulk flow γ toward us ~~is~~ are

$$P(\gamma_e) = \frac{4}{3} \sigma_T c \gamma^2 \gamma_e^2 \frac{B^2}{8\pi} \quad \text{and} \quad \nu(\gamma_e) = \gamma \gamma_e^2 \frac{q_e B}{2\pi m_e c}$$

The spectral power P_ν (erg/sec/Hz) is $P_\nu \propto \nu^{1/3}$

for $\nu < \nu(\gamma_e)$ and exponentially suppressed for $\nu > \nu(\gamma_e)$. The power peaks at $\nu(\gamma_e)$ at

$$P_{\nu,max} \simeq \frac{P(\gamma_e)}{\nu(\gamma_e)} = \frac{m_e c^2 \sigma_T}{3 q_e} \gamma B$$

So far, we have assumed that the ~~power~~ energy radiated by an e^- is small compared with its energy, but this will not be valid above a critical γ_c set by the condition

$$\gamma \gamma_c m_e c^2 = P(\gamma_c)\, t \Rightarrow \gamma_c = \frac{6\pi m_e c}{\sigma_T \gamma B^2 t} = \frac{3 m_e}{16 \epsilon_B \sigma_T m_p c\, t \gamma^3 n}$$

where t is the post-GRB observer time.

Consider an e^- with initial $\gamma_e > \gamma_c$. This e^- cools to γ_c in time t. As it cools, $\nu \propto \gamma_e^2$, while $E_e \propto \gamma_e$, so ~~the~~ $P_\nu \propto \nu^{-1/2}$ in the range $\nu(\gamma_c) \equiv \nu_c < \nu < \nu(\gamma_e)$. Therefore, an e^- with $\gamma_e > \gamma_c$ initially radiates

$$P_\nu \propto \begin{cases} \nu^{1/3} & \nu < \nu_c \\ \nu^{-1/2} & \nu_c < \nu < \nu(\gamma_e) \\ e^{-\nu/\nu(\gamma_e)} & \nu > \nu(\gamma_e) \end{cases}$$

$P_{\nu,max}$ now occurs at $\nu = \nu_c$

This description is valid for (1) e^- injection is instantaneous and observations averaged over time t or (2) e^- injection is continuous over time t and the observation is instantaneous $\Longleftrightarrow$ GRBs

So far, we have considered just one e^-. We now have to integrate over $N(\gamma_e) \propto \gamma_e^{-p}$.

If $\gamma_m > \gamma_c$ (fast cooling), all e^-'s cool to γ_c, and ~~flux at~~ power at ν_c is $N_e P_{\nu,max}$. Then,

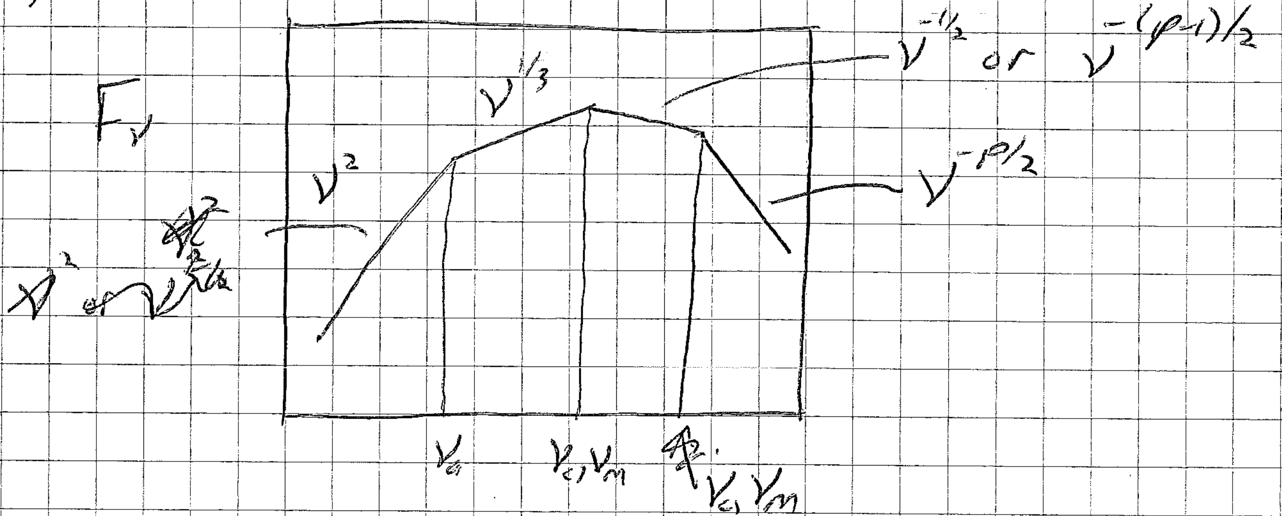
observed flux
$$F_\nu = \begin{cases} (\nu/\nu_c)^{1/3} F_{\nu,max} & \nu < \nu_c \\ (\nu/\nu_c)^{-1/2} F_{\nu,max} & \nu_c < \nu < \nu_m \\ (\nu_m/\nu_c)^{-1/2} (\nu/\nu_m)^{-p/2} F_{\nu,max} & \nu > \nu_m \end{cases}$$

where $\nu_m = \nu(\gamma_m)$ and $F_{\nu,max} \equiv N_e \dfrac{P_{\nu,max}}{4\pi D^2}$

If $\gamma_c > \gamma_m$, only e^-'s with $\gamma_e > \gamma_c$ cool (slow-cooling).
Then,

$$F_\nu = \begin{cases} (\nu/\nu_m)^{1/3} F_{\nu,max} & \nu < \nu_m \\ (\nu/\nu_m)^{-(p-1)/2} F_{\nu,max} & \nu_c < \nu < \nu_m \\ (\nu_c/\nu_m)^{-(p-1)/2} (\nu/\nu_c)^{-p/2} F_{\nu,max} & \nu > \nu_c \end{cases}$$

At even lower frequencies, ~~$F_\nu \propto \nu^x$~~ $\nu < \nu_a$, the synchrotron radiation becomes self-absorbed and $F_\nu \propto \nu^2$:



Self-absorption occurs at low ν because the source-frame specific intensity cannot be higher than blackbody,

$$I_{\nu,BB,e} \overset{RJ}{=} \frac{2\nu_e^2}{c^2} m_e c^2 \gamma_e$$ taking $kT_{eff} \simeq \gamma_e m_e c^2$

Since I_ν/ν^3 is invariant, observer sees maximum,

$$I_{\nu,BB} = \frac{2\nu^2}{c^2} m_e c^2 \gamma_e \gamma.$$

Observed size of emitting ~~area~~ region is $\frac{R}{\gamma}$ and its angular size is $R/\gamma D$, so the observer flux is limited by

$$F_{\nu,BB} = I_\nu d\Omega = \frac{2\nu^2}{c^2} m_e c^2 \gamma_e \gamma^{-1} \left(\frac{R}{D}\right)^2.$$

The afterglow light curves (temporal evolution of F_ν at some frequency ν) are obtained by considering time evolution of γ_c, $N_e P_{\nu,max}$, etc.

The total # of e^-'s in the post-shock fluid is $N_e = 4\pi R^3 \frac{n}{3}$.

Assume that the shock evolution is adiabatic (i.e., that radiative losses are negligible). Then the shock energy E is constant and $E \sim \gamma^2 R^3 n m_p c^2$. Using $t \sim R/\gamma^2 c$,

$$R(t) \simeq \left(\frac{17 E t}{4\pi m_p n c}\right)^{1/4} \qquad \gamma(t) \simeq \left(\frac{17 E}{1024 \pi n m_p c^5 t^3}\right)^{1/8}.$$

If so, then

$$\nu_c = 2.7 \times 10^{12}\, \epsilon_B^{-3/2}\, E_{52}^{-1/2}\, n_1^{-1}\, t_d^{-1/2}\ \mathrm{Hz}$$

$$\nu_m = 5.7 \times 10^{14}\, \epsilon_B^{1/2}\, \epsilon_e^{2}\, E_{52}^{1/2}\, t_d^{-3/2}\ \mathrm{Hz}$$

$$F_{\nu,max} = 1.1 \times 10^{5}\, \epsilon_B^{1/2}\, E_{52}\, n_1^{1/2}\, D_{28}^{-2}\ \mu\mathrm{J}$$

where $t_d = (t/\mathrm{day})$, $E_{52} = E/10^{52}\,\mathrm{erg}$, $n_1 = \frac{n}{\mathrm{cm}^{-3}}$

and $D_{28} = D/10^{28}\,\mathrm{cm}$

The transition from fast to slow cooling occurs when $\nu_c = \nu_m$, which occurs at

$$t = 210\, \epsilon_B^{2}\, \epsilon_e^{2}\, E_{52}\, n_1\ \mathrm{days}.$$

The time evolution of ν_c, ν_m, $F_{\nu,max}$ can give the light curve at a given frequency. E.g., at $\nu = 10^{15}\,\mathrm{Hz}\ \nu_{15}$. The frequencies ν_c, ν_m equal ν when

$$t_c = 7.3 \times 10^{-6}\, \epsilon_B^{-3}\, E_{52}^{-1}\, n_1^{-2}\, \nu_{15}^{-2}\ \mathrm{days}$$

$$t_m = 0.69\, \epsilon_B^{1/3}\, \epsilon_e^{4/3}\, E_{52}^{1/3}\, \nu_{15}^{-2/3}\ \mathrm{days}$$

Since $\nu_c = \nu_m$ at time t_0, we have either

$$t_0 > t_m > t_c \quad \text{or} \quad t_0 < t_m < t_c.$$

Defining $\nu_0 = \nu_c(t_0) = \nu_m(t_0)$, we have

$$\nu_0 = 1.8 \times 10^{11}\, \epsilon_B^{-5/2}\, \epsilon_e^{-1}\, E_{52}^{-1}\, n_1^{-3/2}\ \mathrm{Hz}$$

For frequencies $\nu > \nu_0$, $t_0 > t_m > t_c$; this case is referred to as the high-frequency light curve.

For frequencies $\nu < \nu_0$, $t_0 < t_m < t_c \Rightarrow$ low-frequency light curve

The time-frequency dependence of the afterglow is written as

$$F_\nu \sim \nu^{-\beta} t^{-\alpha}$$

If $\beta = p/2$ (as is case for both slow- and fast-cooling), then $\alpha = 3(p-1)/4$. If for some reason $\beta = (p-1)/2$ (which is what you get for a non-cooling population of e^-'s — e.g., slow-cooling with $\nu_c < \nu < \nu_m$), then $\alpha = (3p/4) - (1/2)$.

Jets and GRB energetics:

GRB energies implied by assuming isotropic emission get up to $\sim 5 \times 10^{54}$ erg $\gg M_\odot c^2$, which is kind of big.

It is now generally accepted that typical GRBs are beamed into a solid angle $\sim 4\pi/100$, and that the energies are $\sim 100\times$ smaller

The evidence comes from breaks in the afterglow power laws. At early times the evolution is that of a jet. At some time t_j when the shock has slowed it begins to spread. The subsequent evolution is thus different, more appropriate for spherical, rather than jet, expansion