Astrophysics of Compact Objects (171.156), Fall 2011

Problem Set 2

Due: In class, 13 September 2011

1. **Convection in a white dwarf envelope**. Consider a white dwarf of $0.6\,M_\odot$, radius $0.01\,R_\odot$ and effective temperature of 12,000 K. A model envelope, assumed to be pure hydrogen with mixing length $l_m = 2H_P$ (i.e., $\alpha = 2$) is found to pass through the following point in the ρ-T plane: $\log T = 4.29$ and $\log \rho = -5.51$ (in cgs units). At that point in the star,

   (a) Calculate the hydrogen ionization fraction.

   (b) Calculate the pressure p. Use this pressure and the equation of hydrostatic equilibrium to calculate the total mass (in units of $M_\odot$) of the envelope above this point.

   (c) Calculate $\nabla_{\rm ad}$.

   (d) Do a google search to find the OPAL opacity tables (should be somewhere at ornl.gov). Find the opacity for pure hydrogen at this temperature and density (this might take some poking around on the OPAL site, but not too much. Keep in mind that the tables give opacities as a function of $\log T$ and $\log R$, where $R = \rho(T/10^6\,{\rm K})^{-3}$.)

   (e) Use the equation of radiative equilibrium to calculate $\nabla_{\rm rad}$. Show that at this point in the star, (at least part of) the flux must be carried by convection.

   (f) Calculate the pressure scale height H_P at this zone, and show that the Rosseland optical depth is $\tau \sim 100$.

   (g) Using the equations in Ch. 7 in KW, Estimate the velocity of convection cells. What is their Mach number?

   (h) Compute ∇. Is the convection efficient or inefficient?

   (i) Estimate the turnover time of the eddies and compare with the ~ 100-sec g-mode period.

2. **Expanding shells in a supernova.** Consider an expanding sphere of initial radius 5×10^8 cm and mass $1.4\,M_\odot$ initially heated to 10^{10} K. Assuming constant density, total ionization, homologous expansion (radial velocity $v \propto r$, where r is the radius), and Thomsen opacity (i.e., $\kappa = 0.2$ cm^2 g^{-1} throughout), calculate the radius the expanding sphere would have when it first becomes optically thin. (As an aside, although the gas in a SN I does not remain fully ionized, the Doppler broadened forest of iron lines provides an opacity comparable to electron scattering.) If the expansion is adiabatic ($\rho \propto T^3$) as well as homologous, what is the

temperature inside the star when it becomes transparent? (Your result should be lower than the actual value because radioactive decays provide some heating.)

3. **Type Ia supernovae.** Consider a model for a Type Ia supernova to be a white dwarf with mass 1.39 $M_\odot$ composed of 50% each (by mass) ^{12}C and ^{16}O. The net binding energy (internal energy plus gravitational potential energy) is -5.0×10^{50} erg. How much energy per unit mass (in erg/g) is released when carbon and oxygen burn to ^{56}Ni? Suppose 0.8 $M_\odot$ of the star burns to ^{56}Ni (for simplicity, neglect the synthesis of intermediate-mass elements here). What will the net energy of the white dwarf be? If it expands to infinity, what would be the typical velocity? (Relevant binding energies: ^{12}C, 92.163 MeV; ^{16}O 127.621 MeV; ^{56}Ni, 484.003 MeV.)

4. **Gamma rays from supernovae.** Suppose that a supernova explosion in Virgo (20 Mpc) produces 0.5 $M_\odot$ of ^{56}Co ($\tau_{1/2} = 77.3$ days). Assuming that after 100 days the supernova becomes transparent to the γ-rays produced during the decay to ^{56}Fe (not true for SN II, but approximately true for SN I), what would be the flux of such γ-rays at the Earth? (The sensitivity limit of the GLAST γ-ray detector is about 10^{-6} photons cm^{-2} sec^{-1}.) Although it is not necessary for your result, each ^{56}Co decay produces gamma rays of 847 keV and 1236 keV.

5. **Supernova classifications.** Check the recent literature to find answers to the following: What are the distinguishing observational features of Type Ia, Ib, Ic, IIp, and II-L supernovae (e.g., spectra, light curves, velocities, location in galaxies, event rates)? What are the leading theoretical models for each? In each, during what epoch(s) is radioactivity an important contributor to the light curve? Keep your answers simple but clear.

6. **Crab-pulsar spindown.** The Crab pulsar is a rotating neutron star formed by a supernova in AD 1054. At present it has an angular velocity and an angular acceleration given by

$$\omega = 190\,\mathrm{sec}^{-1} \quad \text{and} \quad \frac{d\omega}{dt} = -2.4 \times 10^{-9}\mathrm{sec}^{-2}. \tag{1}$$

If gravitational radiation were responsible for the Crab slowdown, the rate of loss of rotational energy would be proportional to ω^6. Use this model to derive an expression for the time dependence of ω and show that this model predicts an age which is less than the actual age of the pulsar. Show that if, instead, the slowdown is due to magnetic-dipole radiation, in which the radiated power is proportional to ω^4, then the predicted age is in good agreement with observations.