Astrophysics of Compact Objects (171.156), Fall 2011

Problem Set 5

Due: In class, 13 October 2011

1. Problem 2.1 in Frank, King, and Raine.

2. **The Trapping Radius in Spherical Accretion** (from Phinney via Bildsten). Consider a purely spherical flow onto a black hole with mass M that dissipates (and radiates) an energy per gram of $\approx GM/r$ as it falls from r to $r/2$. Even though clearly some of the energy must be escaping as radiation, presume that the matter still is roughly falling at the free-fall speed all the way, as we outlined in the opening paragraph at top.

   (a) Calculate the optical depth to Thomson scattering from an inner radius r to infinity $\int \sigma_{Th} n_e dr$ as a function of the accretion rate.

   (b) At what accretion rate (call this $\dot{M}_c$) does the optical depth become unity at $r_g = 2GM/c^2$? How does $\dot{M}_c$ relate to $\dot{M}_{Edd}$?

   (c) For $\dot{M} > \dot{M}_c$, the innermost parts of the flow becomes optically thick and the flow just takes in all the internal energy. The black holes goes gulp! Calculate the "trapping radius", r_t, as a function of $\dot{M}/\dot{M}_c$.

   (d) What is the luminosity, L, that escapes to infinity in the regime $\dot{M} > \dot{M}_c$? How does the efficiency, η, depend on the ratio $\dot{M}/\dot{M}_c$?

   (e) Find L and $\dot{M}_c$ for a $10^8\,M_\odot$ black hole.

3. **Stellar spindown due to Keplerian disk** (from Bildsten adapted from FKR 4.1). Imagine a Keplerian disk where matter is neither expelled nor accreted, but simply extracts angular momentum from the central star at a constant rate $N_o = I\dot{\omega}$, where ω is the central star's spin frequency and I is the stellar moment of inertia.

   (a) Show that at a distance far from the stellar surface, the flux from a surface of the disk is

   $$F(r) = \frac{3N_o(GM)^{1/2}}{8\pi r^{7/2}}, \quad (1)$$

   where M is the stellar mass.

   (b) Integrate this flux up to the stellar surface so as to get the total luminosity. How does it compare to the rotational energy loss of the central star, $L_{rot} = I\omega\dot{\omega}$? Is it more or less? Does the ratio of the luminosities depend in some simple way on the rotation rate of the central object, ω. Discuss the energy balance and what assumptions might go bad as one gets close to the star.

(c) Again, focus on the region at large radii where the simple equation 1 should be adequate. Start by replacing N_o with L_{rot} and then fully work out an α disk model presuming that Kramer's opacity predominates and that the pressure is that of a completely ionized ideal gas. Do this for a neutron star with $M = 1.4M_\odot$ and scale your solution with a fiducial $L_{rot} = 10^{36}$ erg s^{-1} and a spin period of one second. Find h, T in the mid-plane, and Σ as a function of radius (for $r > 100$ km).

(d) Given the density and temperature in the mid-plane, *estimate* the radius at which the hydrogen becomes neutral. How does this position depend on L_{rot}?

4. **Irradiated thin disk.** Here's a simplified version of FKR 5.3. Consider a spherical star of radius R_* and uniform temperature T_* radiating like a blackbody. Assume the star is surrounded by an infinitesimally thin disk of optically thick material. Show that the temperature $T(r)$ of this "passive" disk scales as $r^{-3/4}$ at large distances r. Compare this temperature distribution with that from a standard thin accretion disk and explain how you would distinguish the two with observations of a young star.

5. Do FKR problem 5.5.