Astrophysics of Compact Objects (171.156), Fall 2011

Problem Set 6

Due: In class, 25 October 2011

1. Problem 5.6 in Frank, King, and Raine.

2. (ST 13.3): A bump is observed in the spectrum of Her X-1 at ~ 58 keV with width ~ 12 keV. This has been interpreted as the photon emitted when an electron makes a transition from the first to the lowest Landau level. Determine the magnetic field implied by the frequency and the temperature implied by the width.

3. (ST 13.10) Consider a binary-star system in which one member undergoes a supernova explosion and ejects a considerable fraction of its mass. Assume that the mass is lost in a time much shorter than the orbital period. Calculate the condition on the fraction of mass lost necessary for the preservation of this binary system, assuming a circular orbit. How is the result changed if the orbit is elliptical?

4. (ST 14.14) **Accretion from stellar wind in Cyg X-1:**

   (a) Estimate the rate of gas accretion onto the compact secondary M_x (the black hole) in Cyg X-1. Assume that accretion is from the stellar wind of the primary with the wind velocity consant and comparable to the escape velocity from the primary surface,

   $$V_w = \eta \left(\frac{2GM_*}{R_*} \right)^{1/2} \qquad \eta = \frac{V_w}{V_{\rm esc}}. \tag{1}$$

   Here $M_* \gg M_x$ and R_* are the mass and radius of the primary. Show that the accretion rate is given by

   $$\begin{aligned} \dot{M} &\simeq \frac{1}{4}\left(\frac{r_a}{a}\right)^2 \frac{V_{\rm rel}}{V_w} \dot{M}_w \\ &\simeq (7 \times 10^{-9}\, M_\odot\, {\rm yr}^{-1}) \xi^2 \eta^{-4} \left(\frac{M_x}{10\, M_\odot}\right)^2 \left(\frac{M_*}{30\, M_\odot}\right)^{-8/3} \\ &\quad \times \left(\frac{R_*}{20\, R_\odot}\right)^2 \left(\frac{P}{5.6\, day}\right)^{-4/3} \left(\frac{\dot{M}_w}{10^{-6}\, M_\odot\, {\rm yr}^{-1}}\right), \end{aligned} \tag{2}$$

   where $\dot{M}_w$ is the steady-state spherical wind ejection rate ($\dot{M}_w = 4\pi\rho r^2 V_w$ =constant), a is the separation between the primary and the secondary, and P is the period of the orbit, assumed circular. (**Hint:** $V_{\rm rel}^2 \simeq V_x^2 + V_w^2$, where V_x is the velocity of the secondary relative to the primary. Note that the normalization of the factors here has been chosen to fit the observations of the Cyg X-1 binary system.)

(b) Show that the normalization of parameters here is consistent with the observed x-ray luminosity of $\sim 5\times10^{37}$ erg s^{-1}, assuming around 10% efficiency of conversion of rest mass to radiation.

(c) Calculate the fraction $\dot{M}/\dot{M}_w$ of emitted gas that actually accretes onto the compact secondary.

5. (ST 14.22) **More on accretion from a wind:** Consider the same system as in the previous problem.

(a) Show that the net angular momentum per unit mass carried by the accreted gas is

$$l = \frac{1}{2} V_x a \left(\frac{r_a}{a}\right)^2. \tag{3}$$

(b) Estimate the ratio of l from spherical-wind accretion to l from Roche lobe overflow.

(c) Assume that the outer edge of the disk forms at the radius r_D at which Eq. (3) equals the angular momentum per unit mass of a gas element in circular orbit about the hole: $l = (GMr_D)^{1/2}$. Calculate r_D/r_I (where r_I is the radius of the innermost stable circular orbit) for the parameters appropriate to the Cyg X-1 binary system.

6. (ST 15.1) Consider a neutral beam of cold plasma, initially moving with velocity v in a field-free region, incident normally to a plane surface, beyond which there is a uniform B field parallel to the plane. Neglect the effect of collisions between particles.

(a) Describe the motion of the electrons and ions as they penetrate the plane surface. In particular, argue that a current will be established parallel to the surface, and estimate the penetration depth of the ions.

(b) Estimate the ion and electron contributions to the surface density $\mathcal{J}$ and the corresponding jump in the magnetic field that the current generates.

(c) Argue from part (b) and the expression,

$$\frac{B^2}{8\pi} = \frac{1}{2}\rho(r_A)v^2(r_A), \tag{4}$$

for the Alfvén radius r_A, that these screening currents induced at the Alfvén surface will cancel the stellar field outside r_A.