Astrophysics of Compact Objects (171.156), Fall 2011

Problem Set 7

Due: In class, 1 November 2011

1. **Steady-State Burning of Pure Helium on a Neutron Star.** This is a problem that I got from Lars Bildsten about the physics associated with Type-I x-ray bursts. The neutron star in a low-mass x-ray binary accretes hydrogen-helium–rich matter from a companion star. This accreted matter winds up getting spread over the surface of the neutron star. As additional layers of matter are accreted, those underneath are compressed and heated. There are several possibilities. For example, there may be slow steady-state burning of hydrogen-helium to heavier elements in these accreted layers. But if the conditions (e.g., accretion rate, composition of accreted matter) are right, the base layers of accreted matter may reach temperatures and densities at which the heating due to nuclear burning outpaces the rate at which the heat can be radiated away; the result is then a runaway thermonuclear explosion that appears to us as a Type-I x-ray burst. The purpose of this problem is to work through some of the relevant physics; you may wish to refer to Bildsten's article (astro-ph/9709094; link from class homepage) as you do this problem.

   We'll consider accretion of pure helium onto a neutron star of mass $M = 1.4M_\odot$ and $R = 10$ km. There are only a few such systems known in our galaxy, most of which burn unstably and give Type I X-ray bursts. We will consider only one reaction, which is helium burning to carbon, for which energy is generated at the rate,

   $$\epsilon_{3\alpha} = 5.3 \times 10^{21} \frac{\rho_5^2}{T_8^3} \exp\left(\frac{-44}{T_8}\right) \text{ergs g}^{-1}\text{ s}^{-1}. \tag{1}$$

   where $\rho_5 = \rho/10^5$ g cm^{-3} and $T_8 = T/10^8$ K. The amount of energy released from $3\alpha \rightarrow^{12}$ C is $E_{3\alpha} = 5.84 \times 10^{17}$ ergs g^{-1}. For the equation of state, presume an ideal gas of fully ionized alpha particles, so that $P = (n_e + n_4)kT$, where $n_e = 2n_4$ is the number density of electrons and n_4 is the number density of α particles.

   Let's find out where and when the helium burns after it enters the atmosphere. Work in plane-parallel coordinates, where $\dot{m}$ is the accretion rate per unit surface area, in units gr cm^{-2}s^{-1}.

(a) The flux leaving the atmosphere is set by heat transport

$$F = -\frac{c}{3\kappa\rho}\frac{d}{dz}aT^4 = \frac{c}{3\kappa}\frac{d}{dy}aT^4, \tag{2}$$

where a is the radiation constant. The material is in hydrostatic balance, so $dP/dz = -\rho g$, where $g = GM/R^2$ is constant. Presume a constant flux and $\kappa = 2m_p/\sigma_{\rm Th} = 0.2\ {\rm cm}^2{\rm g}^{-1}$ to show that this equation integrates to

$$T^4 = \frac{3\kappa PF}{acg}, \tag{3}$$

at pressures large compared to the photospheric pressure.

(b) If the fuel is burning in steady-state, then what is the flux leaving the top of the atmosphere as a function of $\dot{m}$?

(c) Use the flux found in part (b) in the equation derived in part (a) to calculate the dependence of temperature on density and/or pressure in the neutron star atmosphere.

(d) The helium burns when the lifetime of the helium nucleus to the 3α reaction ($t_{burn} = E_{3\alpha}/\epsilon_{3\alpha}$) is comparable to the time it takes to get to that depth, $t_{accr} = P/g\dot{m}$. Using the relations found previously, solve the resulting transcendental equation for the temperature where the burning occurs when $\dot{m} = 10^5{\rm gr\ cm}^{-2}\ {\rm s}^{-1}$. What is the pressure at the depth of the helium burning? How long does it take for the matter to reach that depth? (**HINT**: In order to get some feeling for things, you can simplify the transcendental by expanding the exponential about a characteristic temperature of $T_8 = 4$.)

(e) Plot the burning temperature, column depth (P/g), t_{accr}, and density as a function of $\dot{m}$ for the range $10^4{\rm gr\ cm}^{-2}\ {\rm s}^{-1} < \dot{m} < 10^6{\rm gr\ cm}^{-2}\ {\rm s}^{-1}$. Are the electrons always non-degenerate for these accretion rates?

(f) If the gas is cool enough, $T_8 < 5$, then the temperature dependence of the triple-alpha reaction is steep enough that there will be explosive burning of helium. If, however, $T_8 > 5$, the helium will burn stably (see, e.g., Section 2.5 in Bildsten's article). What accretion rate does $T_8 > 5$ require in units of the local Eddington rate?

(g) Observationally, magnetic X-ray pulsars never show evidence for X-ray bursts. This is most likely because the local accretion rate at the polar caps exceeds the critical value found in (f) and that the burning occurs while the matter is still at the polar cap. This means that the accretion flow must only be over a small fraction of the neutron star area. What fraction is that for a neutron star with a total accretion luminosity of $L = 10^{36}$ erg ${\rm s}^{-1}$?

2. Do Frank, King, and Raine's problem 7.2.

3. Do Frank, King, and Raine's problem 7.3.

4. Do Frank, King, and Raine's problem 8.1.