Astrophysics of Compact Objects (171.156), Fall 2011

Problem Set 9

Due: 15 November 2011

1. What is the condition on the energies ϵ_1 and ϵ_2 of two photons that collide at an angle θ if they are to produce an electron-positron pair?

2. A fireball has a rest mass M (baryons) and energy $E = \eta M$. It begins as a sphere with size R_0. What is the requirement on η so that most of the energy will be converted to kinetic energy of the baryons. (The baryons must be coupled to the photons at least up to the time where the Lorentz factor has saturated.)

3. Write the radius at which the shell will begin to spread (increase its width) as a function of the initial fireball parameters E, M, and R_0.

4. Two shells of similar mass and Lorentz factor γ_1 and γ_2 collide. Use conservation of energy and momentum to find the fraction of energy that will thermalize.

5. Write the expression for the synchrotron self-absorption frequency ν_a as a function of time t, fireball energy E, ISM particle density n, and the fractions ϵ_e and ϵ_B of the fireball energy converted to electron energy and magnetic fields. Separate two cases of fast and slow cooling to determine the effective temperature of the radiating electrons.

6. If the flow is not spherical but occupies a cone with opening angle θ, then the evolution is similar to spherical as long as $\gamma > 1/\theta$. After that the jet spreads with $\theta(t) \sim 1/\gamma$. When is the deviation from spherical behavior expected if the opening angle of the jet is $\theta = 0.1$? Use the conservation of energy to show that the radius as function of observed time is almost constant. Find ν_c, ν_m, ν_a and F_{max} for this case. What will be the typical decay rate of the optical flux?