the PN ~~from~~ nucleus continues to shine from shell burning until the shell decreases to $\sim 10^{-3}$–$10^{-4}\,M_\odot$.

The PN expands at a rate ~~$\sim$10 km/s~~ $\sim 10^6$ cm/s, and thus is dispersed after $\sim 10^{4-5}$ yrs

Further evolution of massive stars ($M_i \gtrsim 10\,M_\odot$):

Subsequent stages of burning proceed (in general) non-explosively, degeneracy being achieved only when an iron core is reached.
There is much mass loss (which is not well understood) so that MS evolutionary paths often converge to those of a $M_i \simeq 30\,M_\odot$ star. Wolf-Rayet stars are $M_i \gtrsim 30\,M_\odot$ super-luminous stars with H-depleted envelopes and current masses that may be only ~ 5–$10\,M_\odot$, ~~[illegible]~~ $\sim$400–600 km/sec outflows, and bright N-rich nebulae.
The luminosity of post-C burning stars remains close to L_{edd}, and evolutionary tracks in the HR diagram criss-cross horizontally as subsequent burning stages are ignited and exhausted.

Core collapse:

After Si$\rightarrow$Fe ceases in the core, the Fe core contracts until it is held up by degeneracy pressure.
However, additional Si burning in a shell dumps more Fe onto the core, and when the Fe-core mass exceeds 1.4 $M_\odot$, it undergoes gravitational contraction.
However, unlike earlier stages of stellar evolution, ~~the reason~~ since Fe-group elements are the most tightly bound, there are no further exothermic reactions to increase the internal kinetic energy to supply increased pressure.
Instead, $\gamma + {}^{56}Fe \longrightarrow 13\,{}^{4}He + 4n$ ~~each~~ absorbs 124.4 MeV of energy and at higher T, $e^- + p \rightarrow n + \nu_e \longleftarrow$ which escapes), further reducing the pressure available to counteract gravity.

The core then collapses almost freely under gravity in a time $t_{FF} \simeq (3\pi/32G\rho)^{1/2} \simeq 10^{-3}$ sec, as soon as the Fe core has reached a density $\sim 10^{9}$ g/cm^3 and ~~[illegible]~~ at $T \sim 10^{10}$ K, as determined before.
At very slightly higher T, $\gamma + {}^4He \rightarrow 2p + 2n$ releasing ~ 25 MeV.
1.4 $M_\odot$ core when dissociated absorbs 4×10^{51} erg from iron photodisintegration and 10^{52} erg by ^4He disintegration, for a total of 1.4×10^{52} erg.
Comparable to the integrated energy output of the sun over 10 billion years: $\cancel{2}\times10^{33} \times 3\times10^{7} \times 10^{10} \simeq \underset{1.2}{\cancel{6}}\times10^{51}$.

At ~~also~~ <u>e^- capture and neutronization</u>:

At sufficiently high ρ, ~~$e^- + p \rightarrow n + \nu_e$~~ the Fermi (total) energy will be $E_F \gtrsim 1.3$ MeV $= M_n - M_p$, and $e^- + p \rightarrow n + \nu_e$ will begin to convert protons to neutrons.
In the iron core, there are no free neutrons, but the process ~~[illegible]~~ $e^- + {}^{56}Fe \rightarrow {}^{56}Mn + \nu_e$ (inverse-beta decay) occurs for $E_F \gtrsim 4.2$ MeV (again, total energy KE $= 3.7$ MeV), which occurs at density $\rho \simeq 1.1\times10^{9}$ g/cm^3.
At $\rho = 1.5\times10^{10}$ g/cm^3 $e^- + {}^{56}Mn \rightarrow {}^{56}Cr + \nu_e$.
At $\rho \simeq 10^{11}$ g/cm^3, $n \rightarrow p$ becomes very rapid and the "neutronization neutrinos" carry energy away, thereby accelerating the collapse.
The typical ν energy will be

$$\frac{E_\nu}{m_e c^2} \simeq \frac{E_F}{m_e c^2} = \frac{p_F}{m_e c} = \left(\frac{3}{8\pi m_p}\right)^{1/3} \frac{h}{m_e c} \left(\frac{\rho}{\mu_e}\right)^{1/3} \simeq 10^{-2} \left(\frac{\rho}{\mu_e}\right)^{1/3}$$

or $E_\nu \sim 10$ MeV for $\rho \sim 2\times10^{13}$ g/cm^3. Since there are $\sim 10^{57}$ nucleons, roughly

$$E_{cap} \simeq 10^{57} (10\times1.6\times10^{\overset{-6}{\cancel{-12}}}) = 1.6\times10^{52} \text{ erg}$$

are carried away by neutronization ν's. As we will see, this is only a small fraction of the total ν flux.
Although the collapse timescale is $\sim 10^{-3}$ sec, the ν burst from SN 1987A lasted ~ 10 secs. This is because at sufficiently high densities, the collapsing core becomes opaque to ν's.

These ν's interact primarily by a coherent scattering from nuclei of mass number A with cross section,

$$\sigma_\nu \simeq 10^{-45}\,\mathrm{cm}^2 \left(\frac{E_\nu}{m_e c^2}\right)^2 A^2 \qquad \text{for } \nu+(Z,A)\rightarrow\nu+(Z,A)$$

$$\simeq A^2\left(\frac{\rho}{\mu_e}\right)^{2/3} 10^{-44}\,\mathrm{cm}^2,$$

and since $n \simeq \rho/(A m_p)$, the mean-free path is

$$l_\nu \simeq \frac{1}{n\sigma_\nu} = \frac{1}{\mu_e A}\left(\frac{\rho}{\mu_e}\right)^{-5/3} 1.7\times10^{25}\,\mathrm{cm}.$$

With $\mu_e \simeq 2$, $A \simeq 100$, $l_\nu = 10^7$ cm (the core radius) for $(\rho/\mu_e) \simeq 3.6\times10^9$ g/cm³. "Detailed calculations" show that ν's are trapped in the collapsing core for $\rho \gtrsim 3\times10^{11}$ g/cm³.

When the core has reached nuclear densities,

$$\rho \simeq \rho_{nuc} \simeq \frac{3 A m_N}{4\pi r_0^3} = 2.3\times10^{14}\,\mathrm{g/cm^3} \quad (\text{using } r_0 = 1.2\times10^{-13}\,\mathrm{cm})$$

the neutrons become degenerate, and their pressure halts further collapse, and a proto-neutron-star of radius $R \sim 2\times10^6$ cm and mass $M \sim 1.4\,M_\odot$ remains. The gravitational energy released in this collapse is

$$\Delta E_{grav} \simeq +GM_c^2/R \simeq 3\times10^{53}\,\mathrm{erg}$$

which is ~20x the amount absorbed by photodisintegration and neutronization.

When this collapse occurs, ~~In~~ the "bounce" drives a shock into the envelope that expels the envelope.

Although the details are not well understood (and probably require neutrino energy transport), observations show that ~10 $M_\odot$ of envelope are ejected at velocities $V \sim 10^9$ cm/sec, implying a kinetic energy,

$$E_{kin} \simeq \frac{1}{2}10M_\odot V^2 \sim 10^{52}\,\mathrm{erg} \quad \ll \Delta E_{grav}$$

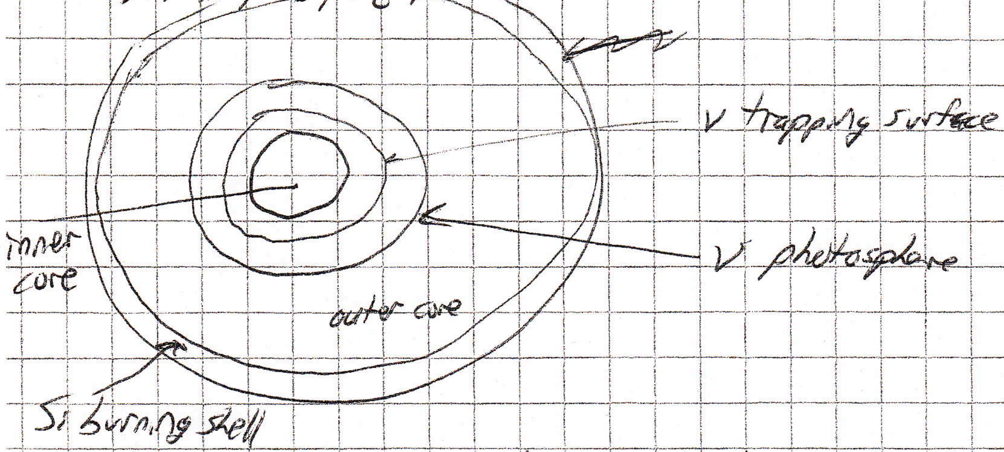
(Note that this is ~~~10~~ 2x the envelope binding energy,

$$E_{bind} \simeq \frac{GM_c(M-M_c)}{R_c \approx 0.01 R_\odot} \sim 5\times10^{51}\,\mathrm{erg}).$$

It is also the gravitational binding energy is also much larger than the optical energy of the SN:

$$E_{opt} \sim L_{SN} \tau_{SN} \sim (10^{44}\,\mathrm{erg/sec})(3\times10^{7}\,\mathrm{sec}) \sim 3\times10^{51}\,\mathrm{erg}$$

Most (≳90%) of the binding energy is liberated as ν's. Neutrinos come into thermal equilibrium in the proto-NS ($\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$) and are emitted from a "neutrinosphere":



We can get a rough understanding of the size of the neutrinosphere by assuming all 6 ν degrees of freedom are emitted thermally from radius R. The luminosity,

$$L_\nu \simeq \frac{3\times10^{53}\,\mathrm{erg}}{10\,\mathrm{sec}} = 3\times10^{52}\,\mathrm{erg/sec}$$

must then be

since ν's are fermions for 6 ν dof as opposed to 2 for γ's

$$L_\nu = 4\pi R_\nu^2 \left(\frac{7}{8}\right)\left(\frac{6}{2}\right)\sigma T_\nu^4$$

$$= 7.5\times10^{53}\,(R/2\times10^{6}\,\mathrm{cm})^2 (T/10^{11}\,\mathrm{K})^4\ \mathrm{erg/sec}$$

For $L_\nu = 3\times10^{52}$ we get reasonable agreement with what we expect for the radius and T (note that the detected ν's had energies ~10 MeV ~ 10^{11} K). Real-world complications include the fact that ν_μ and ν_τ have different interaction strengths and so are emitted at higher T_ν from smaller R; also the spectrum may not be precisely thermal; and the time evolution changes T_ν, etc on a ~10-sec timescale.

As the shock generated by core collapse propagates through the mantle, it heats the material to $T > 5\times10^9$ K for layers below Ne-O layer. At those T, NSE is achieved in a few seconds (the dynamical time) producing primarily ^{56}Ni. The ejected ^{56}Ni than decays,

$$^{56}\mathrm{Ni} \xrightarrow[3\times10^{16}\,\mathrm{erg/g}]{\beta\text{-decay}} {}^{56}\mathrm{Co} \xrightarrow[6.4\times10^{16}\,\mathrm{erg/g}]{\beta\text{-decay}} {}^{56}\mathrm{Fe}$$

$$\tau_{1/2} = \underset{6d}{\cancel{\text{[illegible]}}}\ \text{days} \qquad \tau_{1/2} = 77.1\ \text{days}$$

This powers the light output of the SN during the decaying part of its light curve.
For SN1987a, ~0.075 $M_\odot$ of ^{56}Ni was ejected.
For Type-I SN, where a degenerate CO is blown up, almost all 1.4 $M_\odot$ are converted to ^{56}Ni.

EGRET on CGRO detected 1.8-MeV γ's from decay of an excited state of ^{26}Al with $\tau_{1/2} = 7.2\times10^5$ yr throughout the Galaxy. This provides evidence for ongoing stellar nucleosynthesis from SN. The decay products of ^{26}Al have been detected in solar-system meteorites indicating the solar system was formed from SN products.

Finally, the gamma-ray burst of 29 March 2003 (and to some extent, 1998bw before it) demonstrates that at least some if not all/most GRBs are SN. The suggestion is that in some SN with rotating progenitors, the core may collapse to a BH rather than a NS, and an accretion disk or torus may form driving jets through the star and producing the GRB along this jet.