White Dwarfs:

These show up in the lower (low-L) left (~~hot~~ cool) corner of the HR diagram with $T_{eff} \sim 5000-8000$ K.

67% are DA — have H lines in absorption
8% DB — no H; only He
14% DC — no lines at all

Have masses $\sim 0.5 M_\odot$ $R \sim 5 \times 10^3$ km (from $L \propto T^4 R^2$)

Infer from P and T at star's center that cannot have H, or else would be much brighter from H burning; and similarly for other reactions — no nuclear burning

WDs are endpoints of $M \lesssim 8 M_\odot$ stellar evolution, what's left over after He → CO in core. Thus, most WDs are composed of CO. (In stars of sufficiently low mass — $\lesssim 0.2 M_\odot$? — He core not dense enough to burn, and so wind up with He core.)

Are compositionally stratified due to high g; in DA, have thin H outer layer, then He, then CO.
In DB/DC, H shell lost? mixing?

Some WDs are variable:

ZZ ceti stars: $T \sim 12{,}000$ K, $P \sim 100-1000$ sec with multiple periods
- are variable DAs (DAVs) with non-radial oscillations driven by H partial ionization

DBV stars: He-ionization driven pulsations @ $T_e \sim 27{,}000$ K
PNNV — planetary nebula variable (birth of WD).

WDs are held up by e^- degeneracy pressure. The ~~[illegible]~~ structure of the WD can be described well as a polytrope, but we will do only simple estimates here.

The e^- density at the center of the WD is $n_e = \frac{Y_e \rho_c}{m_H}$, where $Y_e \simeq 0.5$ is the # e^-'s per nucleon.
The e^- degeneracy pressure is

$$P = A n_e^{5/3} = A\left[\frac{Y_e \rho_c}{m_H}\right]^{5/3}. \qquad A = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3}$$

(This comes from $P = \frac{g}{(2\pi)^3}\int_0^{p_F} \frac{p^2}{2E}\, d^3p \propto p_F^5$,

and $p_F \propto n_e^{1/3} \sim (\Delta x)^{-1/3}$).

From the eqn of hydrostatic equilibrium, $\frac{dP}{dr} = \frac{-G m(r) \rho(r)}{r^2}$, and noting that ~~M~~ $R \propto (M/\rho)^{1/3}$, we estimate the central pressure to be

$$P_c = A\left[\frac{Y_e \rho_c}{m_H}\right]^{5/3} = B G M^{2/3} \rho_c^{2/3},$$

where $B \sim 1$ (e.g., $0.48 = B$ for $n = \frac{3}{2}$ polytrope, and $B = 0.36$ for $n = 3$; $B \leq 0.8$ most generally),

$$\rho_c = \frac{3.1}{Y_e^3}\left(\frac{M}{M_*}\right)^2 \frac{m_H}{(h/m_e c)^3}\left(\frac{B}{0.44}\right)^3$$

where $M_* = \alpha_G^{-3/2} m_H = 1.85\, M_\odot \qquad \alpha_G = \frac{G m_H^2}{\hbar c} = 5.9\times 10^{-39}$

Since $\rho_c \propto M/R^3$, where R is the radius, WDs have

$$M \propto \frac{1}{R^3} \quad \text{or} \quad R \propto M^{-1/3};$$

they get smaller as $M \uparrow$!!!

Thus, as $M\uparrow$, ρ_c increases, and since $p_F \propto n_e^{1/3}$, at some point, will have $p_F \sim m_e c$, at which point the e^-'s are no longer NR.

The e^-'s will be relativistic if $\rho_c \gtrsim \frac{m_H}{(h/m_e c)^3}$.

Numerically, this occurs when $\frac{3.1}{(1/2)^3}\left(\frac{M}{1.85 M_\odot}\right)^2 \sim 1$, or when

$$M \sim \left(\frac{1}{24}\right)^{1/2} 1.85\, M_\odot \sim 0.4\, M_\odot.$$

Thus, even $M_\odot \sim 0.4\, M_\odot$ is on the hairy edge of validity.

At higher densities, we must use the relativistic EOS,

$$P = B' n_e^{4/3} = B'\left(\frac{Y_e \rho_c}{m_H}\right)^{4/3} \qquad B' = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3}$$

We then get from hydrostatic equil,

$$B'\left(\frac{Y_e \rho_c}{m_H}\right)^{4/3} \simeq B\, G M^{2/3} \rho_c^{4/3}.$$

Note that ρ_c cancels out here. More carefully, we have P somewhere between $\rho^{5/3}$ and $\rho^{4/3}$. As $M\uparrow$, $\rho_c\uparrow$, and these eqns imply that $\rho_c \to \infty$ when $M \to M_{ch}$, with

$$M_{ch} \simeq \left(\frac{B}{0.44}\right)^{-3/2}\left(\frac{Y_e}{m_H}\right)^2\left(\frac{C'}{G}\right)^{3/2}$$

$$\cancel{\simeq 4.8 \left(\frac{B}{0.44}\right)}$$

If we use a polytrope for $P \propto \rho^{4/3}$ $(n=3)$, $B \simeq 0.36$, and

$\underline{M_{ch} \simeq 1.4\, M_\odot}$. <u>Chandrasekhar mass</u>

If WD gains enough matter to drive it to $M \gtrsim 1.4 M_\odot$, it rapidly collapses to a point; e.g. core-collapse SN if Fe WD is at center of $M \gtrsim 8 M_\odot$ star.

Back to M-R relation:

In low-mass regime, where $P \propto \rho^{5/3}$ star is a polytrope with $n = 3/2$, which has central density $\rho_c \simeq 6 \langle \rho \rangle$.

$$R = \left(\frac{3M}{4\pi\langle\rho\rangle}\right)^{1/3} \simeq 0.77\, Y_e^{5/3} \left(\frac{M_\odot}{M}\right)^{1/3} \alpha_G^{-1/2} \frac{\hbar}{m_e c}$$

$$\simeq \frac{R_\odot}{74}\left(\frac{M_\odot}{M}\right)^{1/3} \simeq 0.0135\left(M_\odot/M\right)^{1/3} R_\odot$$

observationally, e.g.,

	$M/M_\odot$	$R/R_\odot$	(predicted)
Sirius B	1.053	0.0074	0.035
40 Eri B	0.48	0.0124	(0.0172)
Stein 2051	0.50	0.0115	

(note that in Fe WD, $Y_e \simeq \frac{26}{56} \gtrsim \frac{6}{12}$, so ~~predts~~ predicts slightly smaller radius)

Using $L = 4\pi R^2 \sigma T_E^4$, find

$$L \simeq \frac{1}{(74)^2}\left(\frac{M_\odot}{M}\right)^{2/3}\left(\frac{T_E}{6000}\right)^4 L_\odot .$$

Thus $L \propto T_E^4$, as seen in HR diagrams.

$\exists$ small scatter, due to scatter in M.

Spread in T_E due to cooling of WDs, with older stars cooler; will see below.

WD has surface gravity $g = \frac{GM}{R^2} \simeq 4\times10^7\ \mathrm{cm/sec^2}$

and redshift $\frac{\Delta\lambda}{\lambda} \simeq \cancel{\hbar\omega} \frac{GM}{R} \simeq 6\times10^{-5}$ for $M \simeq 0.48 M_\odot$

(~~as opposed~~ to 8Å (consistent with measurements))

Cooling of White Dwarfs:

WD is born when He burning ends, so has initially $T \sim 10^8 K$. Degenerate e^-'s conduct heat very efficiently, so core has uniform temperature. H atmosphere has, however, high opacity and thus insulates the core.

Let's consider the H atmosphere. We'll assume that its thin compared with R, and has negligible mass. Then,

$$\frac{dP}{dr} = \frac{-GM\rho(r)}{r^2} \qquad \frac{dT}{dr} = -\frac{3\rho(r)\kappa(r)}{4ac[T(r)]^3}\frac{L}{4\pi r^2}$$

opacity

hydro equil   eqn. of radiative transport

Combined, $$\frac{dP}{dT} = \left(\frac{16\pi ac\,GM}{3L}\right)\frac{T^3}{\kappa}$$

Use Kramer's opacity (for 90% He, 10% heavier),

$$\kappa = \kappa_0 \rho T^{-3.5} = 4.34\times10^{20}\,\rho T^{-3.5}\ cm^2/g$$

Using $P = \frac{\rho kT}{\mu}$, $\kappa = \left(\frac{\kappa_0 \mu}{k}\right) P T^{-4.5}$, or

$$\frac{dP}{dT} = C\frac{T^{7.5}}{P} \quad \text{with} \quad C = \left(\frac{16\pi ac\,Gk}{3\kappa_0 \mu}\frac{M}{L}\right)$$

Integrating with $P(T=0)=0$,

$$\frac{P^2}{2} = C\frac{T^{8.5}}{8.5}$$

In He atmosphere, e^-'s provide $\frac{2}{3}$ of pressure, so

$$n_e = \frac{2}{3}\frac{P}{kT} = \frac{2}{3k}\left(\frac{C}{4.25}\right)^{1/2} T^{13/4}.$$

The atmosphere meets the core when $n_e \simeq 10\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}$.

Find for the isothermal interior T_I

$$T_I \simeq (7\times10^7 K)\left(\frac{L/L_\odot}{M/M_\odot}\right)^{2/7},$$

So it has luminosity,

$$L \simeq \left(\frac{T_I}{7\times10^7 K}\right)^{7/2}\left(\frac{M}{M_\odot}\right)L_\odot$$

The internal thermal energy stored in WD's ions is

$$E \simeq \frac{3}{2}NkT_I = \frac{3}{2}\left[\frac{M}{12 m_H}\right]kT_I$$

$$\simeq 8\times10^{47} \text{ erg for } M \simeq 0.4 M_\odot \text{ @ } T \simeq 10^8 K$$

(more realistically, as WD cools to lattice, $\frac{3}{2}NkT \to 3NkT$)

The cooling rate is then

$$\frac{dT_I}{dt} = -\alpha\left(\frac{T_I}{7\times10^7 K}\right)^{7/2} \text{ with } \alpha \simeq \frac{2}{3k}\left(\frac{12 m_H}{M_\odot}\right)L_\odot$$

$$\simeq 6 K/yr$$

Can integrate to get $T(r)$; e.g., takes ~Gyr for $0.4 M_\odot$ $T\sim10^8 K$ WD to cool from $L_\odot$ to $10^{-4} L_\odot$.