Quantum Mechanics (171.605), Fall 2016

Problem Set 9

Due: 13 December 2016

1. Problem 4.11 in Sakurai-Napolitano.

2. Problem 4.12 in Sakurai-Napolitano.

3. Determine the energy levels for a particle of mass m moving in one-dimensional lattice of Dirac delta functions; i.e.,

$$V(x) = \sum_{n=-\infty}^{\infty} V_0 \delta(x - na). \tag{1}$$

4. The Hamiltonian of a positronium atom in the $1S$ state in a magnetic field B along the $\hat{z}$ axis is to good approximation,

$$H = A\vec{S}_1 \cdot \vec{S}_2 + \frac{eB}{mc}(S_{1_z} - S_{2_z}), \tag{2}$$

if all higher energy states are neglected. The electron is labeled as particle 1 and the positron as particle 2. Using the *coupled representation* in which $\vec{S}^2 = (\vec{S}_1 + \vec{S}_2)^2$ and $S_z = S_{1_z} + S_{2_z}$ are diagonal, obtain the energy eigenvalues and eigenvectors and classify them according to the quantum numbers associated with the constants of the motion. Empirically it is known that for $B = 0$ the frequency of the $1^3S \to 1^1S$ transition is 2.0338×10^5 MHz and that the mean lifetimes for annihilation are 10^{-10} sec for the singlet state (two-photon decay) and 10^{-7} for the triplet state (three-photon decay). Estimate the magnetic-field strength B which will cause the lifetime of the longer lived $m = 0$ state to be reduced (or "quenched") to 10^{-8} sec.

5. The spin-angular function for a spin-1/2 particle in a state of total angular momentum $j = l \pm (1/2)$ and azimuthal quantum number m is

$$\mathcal{Y}_l^{l\pm(1/2),m} = \frac{1}{\sqrt{2l+1}} \begin{pmatrix} \pm\sqrt{l \pm m + \frac{1}{2}}Y_{l,m-(1/2)} \\ \sqrt{l \mp m + \frac{1}{2}}Y_{l,m+(1/2)} \end{pmatrix}. \tag{3}$$

Now suppose that this spin-1/2 particle is in a state,

$$\mathcal{Y}_{j-(1/2)}^{jm} + b\mathcal{Y}_{j+(1/2)}^{jm}, \tag{4}$$

with total angular momentum jm. Assume this state to be an eigenstate of the Hamiltonian H with no degeneracy other than that demanded by rotational invariance. If H conserves parity, how are the coefficients a and b restricted? If H is invariant under time reversal, show that a/b must be imaginary. Verify explicitly that the expectation value of the dipole operator $-e\mathbf{r}$ vanishes if either parity is conserved or if time-reversal invariance holds (or both).