Particle Astrophysics (171.697), Spring 2017

Problem Set 1

Due: In class, first class of week 2

1. Find a coordinate transformation that shows that the Milne spacetime (i.e., the FRW spacetime with Friedmann equation $H^2 \propto a^{-2}$) is equivalent to a Minkowski spacetime. Explain what is going on.

2. The age t_0 of the Universe in the standard cosmological model depends on the current value of the Hubble parameter, $H_0 = 100\,h$ km/sec/Mpc, as well as on Ω_0, the current density in units of the critical density. In class, we showed that if $\Omega_m = 1$ and $\Omega_\Lambda = 0$, then $t_0(h) = 6.7\,h^{-1}$ Gyr. (a) Your assignment is to generalize this result and derive an expression for the age of the Universe for $\Omega_m > 1$ and for $\Omega_m < 1$, both for $\Omega_\Lambda = 0$. (This shouldn't be too tricky—the answers are in the books. But still, you should derive the equations yourself.) Then, plot contours for t_0 =10 Gyr, 13 Gyr, and 17 Gyr on the $\Omega_m - h$. You may do this either by sketching the contours by hand, or you may generate such a plot with Mathematica, C, Fortran, or anything else, if you're so inclined. (b) Then, make the analogous plots, but for $\Omega_m + \Omega_\Lambda = 1$ (and restricting to $\Omega_m < 1$). (c) Stellar astrophysicists believe that the oldest stars are around 10–20 Gyr. If the correct value is somewhere around 14 Gyr, your plots should show you for which values of Ω_m and h there might be consistency. (d) At some point, astronomers found a galaxy at a redshift $z = 1.5$ with a spectrum well fit by stellar-population model with an age 3.5 Gyr. Draw on your plots the regions of the Ω_m-h parameter space ruled out by this measurement.

3. The brightness of sources are measured on a logarithmic *apparent magnitude* scale, where the apparent magnitude is defined to be $m = -2.5 \log f +$ constant, and f is the flux, and the logarithm is base 10. The luminosity of the source is similarly measured on a logarithmic *absolute luminosity* scale, where the absolute luminosity is $M = -2.5 \log L +$constant. The constants are chosen so that the *distance modulus* is $m - M = 5 \log(r/10\,\mathrm{pc})$. If there is a *standard candle*, an object of known luminosity L (and thus known M, then measurement of its apparent magnitude m determines the luminosity distance. Suppose now that observers measure the distance modulus of supernovae (assumed to be standard candles) at redshifts $z = 0.5$ and $z = 1$. Calculate the luminosity distances at these two redshifts for (i) an (ii) a flat Universe with nonrelativistic matter and a cosmological constant with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$; and (ii) the same Universe with the cosmological constant replaced by some exotic dark energy with equation-

of-state parameter $w_{\rm DE} = -0.9$ and with $\Omega_m = 0.3$ and $\Omega_{\rm DE} = -0.9$. Determine the differences in the distance moduli between these two cosmologies, both at redshifts $z = 0.5$ and $z = 1$. Do some reading or talk with some of the local advanced grad students, professors, or postdocs to figure out how well (roughly) the magnitude scale is calibrated and also to figure out what a typical extinction is and how accurately it can be subtracted from reddening measures.

4. In the standard cosmological scenario, all the electrons and protons in the Universe combine to form hydrogen at a redshift $z \simeq 1000$ in a process we call "recombination". However, at some redshift $z_{\rm reion}$, stars begin to form and emit radiation that ionizes all the hydrogen in the Universe. If so, then cosmic microwave background (CMB) photons may Thomson scatter from the free electrons en route from the surface of last scatter. Calculate the optical depth $\tau(z_{\rm reion})$ for Thomson scattering of CMB photons as a function of the reionization redshift $z_{\rm reion}$ for $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Derive an analytic approximation for $z_{\rm reion} \gg \Omega_m^{-1}$ and make sure your exact expression agrees with this answer in the appropriate limit. Write your answer in terms of the baryon density $\Omega_b h^2 \simeq 0.022$ (where h is the Hubble parameter in units of 100 km/sec/Mpc) and in terms of the helium mass fraction $Y \simeq 0.24$. At what $z_{\rm reion}$ does $\tau = 1$?

5. When a supernova goes off, it injects roughly 10^{51} ergs of kinetic energy into the surrounding interstellar medium (ISM), thus driving a shock wave into the ISM that then heats the ISM. The heated gas then cools primarily by Brehmsstrahlung emission. This is a complicated process and can differ significantly from one supernova to another. Still, for the purposes of this exercise, let's suppose that the cooling time for a typical supernova remnant is 10,000 years. To a much lesser degree, the electrons in the gas can also cool by inverse-Compton scattering cosmic microwave background (CMB) radiation. Today, (at redshift $z = 0$) the CMB has a temperature $T = 2.7$ K. However, at earlier times, at redshift z, the energy density of the CMB will be much higher and the efficiency of inverse-Compton cooling of the supernova remnant will be much higher. *Estimate* the redshift at which inverse-Compton cooling becomes the primary avenue for the supernova remnant to cool. (To do so, you may need to look up the formula for the Compton cooling rate—no need to derive it yourself.) If you do this sufficiently carefully, you could write an *ApJ* article about it; if so, you are spending too much time on this problem. We're just looking here for a back-of-the-envelope estimate, which should not be too difficult.