# Particle Astrophysics (171.697), Spring 2017

### Problem Set 12

### Due: In class, first class of week 13

1. Suppose that we make $N$ measurements $x_i$, each labeled $i$ (with $i = 1, \ldots, N$), of a Gaussian variable $x$ with zero mean ($\langle x \rangle = 0$) and from these we want to determine the variance $\langle x^2 \rangle = \sigma^2$ of the distribution. Show that the variance with which we measure $\sigma^2$ is $\Delta\sigma^2 = (2/N)^{1/2}\sigma^2$. How does this result change if each measurement $x_i$ is obtained with a standard error $\delta x$.

2. We showed in class that the convergence power spectrum is

$$C_\ell^{\kappa\kappa} = \int_0^{D_s} dD_l \, \frac{W^2(D_l, D_s)}{D_l^2} P\left(\ell/D_l; t(D_l)\right), \tag{1}$$

for sources at a distance $D_s$. Here,

$$W(D_l, D_s) = \frac{3\Omega_m H_0^2}{2c^2 a(D_l)} \frac{D_{sl} D_l}{D_s}, \tag{2}$$

is the lensing window function in terms also of the lens distance $D_l$, matter density $\Omega_m$, Hubble parameter $H_0$, speed of light $c$, and scale factor $a(D_l)$ (where here the scale factor is evaluated at the age of the Universe at the time that a photon we observe today would have passed a distance $D_l$. Also, $D_{sl}$ is the lens-source distance, and all distances are angular-diameter distances. Also, note that the power spectrum must be evaluated at the time corresponding to a distance $D_l$; i.e., the matter power spectrum is $P\left(\ell/D_l; t(D_l)\right) = P\left(\ell/D_l; t_0\right) \left[F\left(t(D_l)\right)\right]^2$ in terms of the linear-theory growth factor $F(t)$ (normalized to $F(t_0) = 1$ today—usually we call this $D(z)$, but I use $F$ instead since $D$s are referring to distance here). Rewrite the equation for $C_l^{\kappa\kappa}$ as an integral over redshift $z$, rather than $D_l$. It should then look like

$$C_\ell^{\kappa\kappa} = \int_0^{z_s} dz \, P\left(\ell/D_l(z)\right) G\left(z_s, z_l\right). \tag{3}$$

Plot $G(z_s, z_l)$ as a function of the lens redshift $z_l$ for sources at redshift $z_s = 1$; redshift $z_s = 3$; and redshift $z_s = 1100$. From these plots you should be able to infer the redshifts probed by cosmic-shear surveys of galaxies at different redshifts and of the CMB.

3. Use CAMB and/or CLASS to plot the lensed and unlensed temperature power spectra to see the effects of lensing. The effects you should see include (a) smoothing of the acoustic peaks and (b) generation of additional power on small scales. Then plot the B-mode power spectrum due to weak lensing. You should find $C_\ell^{\rm BB}$ from lensing to behave like $\ell^2 C_\ell \propto \ell^2$ at $\ell \ll 1000$. Try to derive this behavior from the analytic integrals for $C_\ell^{\rm BB}$ in terms of the primordial E-mode power spectrum and the convergence power spectrum. This low-$\ell$ behavior resembles that from instrumental noise. What value of the instrumental noise (or noise-equivalent temperature (NET), specified in units of $\mu{\rm K}^2/{\rm sec}$) would give, in an all-sky survey, the same amplitude for the low-$\ell$ $C_\ell^{\rm BB}$?