Particle Astrophysics (171.697), Spring 2017
Problem Set 3
Due: first class of week 4

1. Consider a critical-density Universe in which massive neutrinos contribute Ω_ν to the density parameter. Show that on scales smaller than the neutrino Jeans length, perturbations in the remaining cold component grow as $\delta \propto t^\alpha$, where $\alpha = (\sqrt{25 - 24\Omega_\nu} - 1)/6$. (Hint: The Ω_ν of the critical density in neutrinos contributes to the expansion rate, but this component remains smoothly distributed.)

2. In this problem you will explore numerically the growth of a spherical perturbation in a cosmological-constant Universe. A spherically-symmetric perturbation collapses in a flat cosmological-constant Universe (i.e., $\Omega_m + \Omega_\Lambda = 1$) of arbitrary Ω_m. Derive an exact density contrast at virialization (you will probably not be able to do this analytically), and compare with the oft-quoted estimate, $1 + \delta = 178\Omega_m^{-0.7}$.

3. Consider linear growth of perturbations in a Universe with cold dark matter with density $\Omega_{\rm cdm} = 0.25$ and baryon density $\Omega_b = 0.05$. Consider only redshifts $z \gg 1$ so that the dynamical effect of the cosmological constant is negligible. Write down the differential equations for linear evolution of $\delta_{\rm cdm}(\vec{x}, t) = \delta\rho_{\rm cdm}(\vec{x}, t)/\bar{\rho}_{\rm cdm}$, the fractional perturbation to the CDM density, and for $\delta_b(\vec{x}, t) = \delta\rho_b(\vec{x}, t)/\bar{\rho}_b$, the fractional perturbation to the baryon density. Now consider the evolution of a single Fourier mode of wavelength λ and wavenumber $k = 2\pi/\lambda$, of the density field. Show that baryon perturbations are stabilized by pressure at small scales, and find an expression for the Jeans wavelength Λ_J, the wavelength that separates stable and unstable modes. Evaluate the Jeans wavelength just before and just after recombination, and determine the corresponding Jeans mass.

4. Calculate the neutrino damping length as a function of the neutrino mass m_ν (for values of m_ν between 0.1 eV and 1 eV). Assume that all three neutrino species have the same mass.