Particle Astrophysics (171.697), Spring 2017
Problem Set 4
Due: first class of week 5

1. Calculate (numerically) and plot the root-variance $\sigma(M)$ as a function of mass M for the ΛCDM power spectrum $P(k)$ for $\Omega_m = 0.3$. Then, determine M_* (defined by $\sigma(M_*, t) = \delta_c$ as a function of redshift from $z = 0$ to $z = 100$. Use the proper linear-theory growth factor (you can use the semi-analytic approximation given in class), and also the proper δ_c (also using the approximation given in class). You can use the ΛCDM transfer function given in class, a more recent semi-analytic version from the literature, or if you're ambitious, obtain it numerically from the publicaly CLASS or CAMB codes.

2. Suppose that the 1-point distribution function, normalized to unit variance is $\mathcal{P}(\nu)$ (e.g., for Gaussian initial conditions, $\mathcal{P}\nu = (2\pi)^{-1/2}\exp(-\nu^2/2)$). (a) Show that the usual Press-Schecter equation for the number of gravitationally-bound halos with masses between M and $M + dM$ per comoving volume at redshift z generalizes to

$$\frac{dn}{dM}dM = \frac{f\rho_b}{M}\mathcal{P}[\nu(M,z)]\frac{\partial\nu(M,z)}{\partial M}dM,$$

where $\nu = \delta_c(z)/\sigma_M$, $\delta_c(z) = 1.69/D(z)$ is the critical overdensity for gravitational collapse, $D(z)$ is the linear-theory growth factor, and σ_M is the variance of the mass distribution for scales M. Also, $f = \int_0^\infty \mathcal{P}(\nu)\, d\nu$. (b) Now suppose that once halos form their mass is fixed, and suppose further that they disappear only when they merge into larger halos. Show that with these assumptions, the distribution (normalized to unity) of formation redshifts z_f for halos of mass M observed at redshift z_0 is

$$\frac{df}{dz_f} = \mathcal{P}'[\nu(M,z_f)]\frac{\partial\nu(M,z_f)}{\partial z_f}\left\{\mathcal{P}[\nu(M,z_0)]\right\}^{-1}.$$

(c) Evaluate this formation-redshift distribution for a Gaussian distribution of perturbations and describe it qualitatively. For hints, see *MNRAS* **321**, L7 (2001).

3. The CMB temperature power spectrum $C_\ell^{\rm TT}$ is defined in terms of the temperature spherical-harmonic coefficients $a_{\ell m}^{\rm T}$ through,

$$\left\langle a_{\ell m}^{\rm T}\left(a_{\ell' m'}^{\rm T}\right)^*\right\rangle = C_\ell^{\rm TT}.$$

The angular temperature two-point correlation function is

$$C^{\rm TT}(\theta) = \langle T(\hat{n})T(\hat{n}')\rangle_{\hat{n}\cdot\hat{n}'=\cos\theta};$$

i.e., it is the expectation value of the product of the temperatures at two points on the sky separated by an angle θ. Derive an expression for the correlation function in terms of the power spectrum, and then the inverse relation for the power spectrum in terms of the correlation function.

4. Consider a vector field V^a that lives on the surface of a two-dimensional sphere. Show that this vector field can be written as

$$V_a = \sum_{\ell=1}^{\infty} \sum_{m=-l}^{l} \left[a^{\mathrm{G}}_{(\ell m)} Y^{\mathrm{G}}_{(\ell m)a} + a^{\mathrm{C}}_{(\ell m)} Y^{\mathrm{C}}_{(\ell m)a} \right],$$

where $Y^{\mathrm{G}}_{(lm)a}$ and $Y^{\mathrm{C}}_{(lm)a}$ are gradient and curl vector spherical harmonics. Derive expressions for these harmonics and show that they are orthonormal. As a preliminary, you can also try this exercise for a vector field on a flat 2d surface.