Particle Astrophysics (171.697) Spring 2012

Problem Set 13

Due: Monday the week after classes end

1. **PPN parameter for Brans-Dicke theory:** In this problem you will calculate the parameterized post-Newtonian (PPN) parameter γ in Brans-Dicke theory in terms of the Brans-Dicke parameter ω, and you will also relate the Newtonian gravitational constant to ω and the value of the Brans-Dicke scalar λ (in Carroll's notation). To do so, consider the weak-field limit of the spherically-symmetric static vacuum spacetime that surrounds a spherical massive body (e.g., the Sun). Thus, write the spacetime metric as
$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2\,d\Omega^2, \tag{1}$$
with $A(r) = 1 + a(r)$ and $B(r) = 1 + b(r)$, where $a(r) \ll 1$ and $b(r) \ll 1$ (and are both linear in the central mass M) in the weak-field limit. You will also need to write the Brans-Dicke scalar as $\lambda = \lambda_0[1 + \epsilon(r)]$, where λ_0 is the value of the Brans-Dicke scalar at large distances, and $\epsilon(r) \ll 1$ (and will also be linear in M). You will then obtain equations of motion for $a(r)$, $b(r)$, and $\epsilon(r)$ by plugging into the Brans-Dicke field equations (for the metric and for the scalar field) retaining terms only to linear order in $a(r)$, $b(r)$, and $\epsilon(r)$. The Newtonian limit is taken by identifying $a(r) = 2GM/r$ (do you know/remember why? if not, think about it), and the PPN parameter γ is defined from $b(r) = 2\gamma GM/r$.

2. **The Schwarzchild spacetime and variational principle:** Show explicitly that the spherically symmetric static vacuum spacetime that minimizes the Einstein-Hilbert action is the Schwarzchild metric. To do so, write the metric in the form,
$$ds^2 = -A(r)\,dt^2 + B(r)\,dr^2 + r^2\,d\Omega^2, \tag{2}$$
and show explicitly that the Einstein-Hilbert action is invariant to any linear variations to $A(r) = (1 - 2M/r)$ and $B(r) = (1 - 2M/r)^{-1}$.

3. (Problem 3.9 in Zwiebach's "A first course in string theory") **Gravitational field of a point mass in a compactified (4+1)-dimensional world:** In this problem you will show that if there is an extra dimension, curled up into a circle of radius a, then at distanves $r \gg a$, the gravitational force due to a point mass decreases as $1/r^2$, but that at distances $r \ll a$, the force law rises (as $r \to 0$) as $1/r^3$. This departure, at small

distance scales, from the $1/r^2$ force law has now been sought in the laboratory (most notably by Eric Adelberger and collaborators in Seattle) at distances smaller than 1mm. OK, Here's the problem: Consider a $(4+1)$-d spacetime with space coordinates (x, y, z, w) *not* yet compactified. A point mass M is located at the origin $(x, y, z, w) = (0, 0, 0, 0)$.

(a) Find the gravitational potential $V_g^{(5)}(r)$. Write your answer in terms of M, the five-dimensional gravitational constant G_5, and $r = (x^2 + y^2 + z^2 + w^2)^{1/2}$. [Hint: Use $\nabla^2 V_g^{(5)} = 4\pi G_5 \rho_m$ and the divergence theorem.]

(b) Now let w become a compact dimension, a circle with radius a, while keeping the mass fixed. Write an exact expression for the gravitational potential $V_g^{(5)}(x, y, z, 0)$. This potential is a function of $R \equiv (x^2 + y^2 + z^2)^{1/2}$ and can be written as an infinite sum.

(c) Show that for $R \gg a$, the gravitational potential takes the form of a four-dimensional gravitational potential, with Newton's constant G_4 given in terms of G_5, as derived in class. [Hint: Turn the infinite sum into an integral.]