Particle Astrophysics (171.697), Spring 2012
Problem Set 3
Due: In class, first class of week 4

Suggested Reading: Ch. 3 in Dodelson's book. Chs. 3–5 in Kolb and Turner's *The Early Universe*. Assorted sections in Peebles' *Principles of Physical Cosmology* cover what we're doing this week, but not in one place. Chs 9–10 in Longair. There may also be relevant parts to Peacock's *Cosmological Physics*.

1. Suppose neutrinos have a mass of 1 eV. Then when their interactions freeze out in the early Universe at a temperature $T \simeq$MeV, they are relativistic, but today they are nonrelativistic. Find the energy distribution $f(K)$ (where K is the kinetic energy) of these cosmological neutrinos today.

2. As discussed in class, BBN can be used as a probe of possible deviations from the standard cosmological model and the standard model of particle interactions. The purpose of this problem is for you to work through some of these constraints. (a) Explain how the primordial ^{4}He abundance would change if the neutron lifetime were longer or shorter. (b) Suppose some particle theorists speculate that Fermi's constant G_F might actually be a function of time. What does BBN constrain the value of Fermi's constant to be at the time of nucleosynthesis? (c) It is plausible that there is a neutrino-antineutrino asymmetry in the Universe, and therefore that the cosmological neutrino mass density *today* is greater than it is in the canonical picture. What is the upper limit to the current neutrino density Ω_ν provided by BBN (assume the neutrinos are massless).

3. Consider a Universe that undergoes anisotropic expansion. Such a Universe has a metric
$$ds^2 = dt^2 - a_x^2(t)\, dx^2 \,+\, a_y^2(t)\, dy^2 \,+ a_z^2(t)\, dz^2,$$
where $a_i(t)$ are scale factors of the three principal axes of the Universe. The Einstein equations for this metric lead to an analog of the Friedmann equation:
$$H^2 \equiv \frac{1}{9}\left(\frac{\dot{V}}{V}\right)^2 = \left(\frac{\dot{\bar{a}}}{\bar{a}}\right)^2 = \frac{8\pi G}{3}(\rho + \rho_s),$$
where ρ is the ordinary energy density (radiation and nonrelativistic matter), and the shear "energy density" is defined to be
$$\rho_s \equiv \frac{1}{48\pi G}[(H_x - H_y)^2 + (H_y - H_z)^2 + (H_x - H_z)^2].$$
Here $V = a_x a_y a_z$ is the "volume scale factor," $\bar{a} = V^{1/3}$ is the mean-scale factor, and the $H_i \equiv (\dot{a}_i/a_i)$ are the expansion rates of the three principal axes. The other Einstein equations (for $i \neq j$) become
$$\frac{d}{dt}\ln|H_i - H_j| = -3H = -3\frac{d}{dt}(\ln \bar{a}),$$

and that this implies that $\rho_s \propto \bar{a}^{-6}$. In other words, the effects of anisotropic expansion are mimicked by a new form of matter with energy density which decreases as $\bar{a}^{-6}$. (a) If ρ_s is too large, it will affect the results of BBN. Find the upper limit to the value of ρ_s *today* provided by BBN. (b) Estimate how this BBN constraint to ρ_s might compare with the CMB constraint that $\Delta T/T < 10^{-5}$. (c) Also, estimate how the BBN constraint might compare with ordinary measurements of the Hubble constant.

4. Determine (a) the physical size of the currently observable Universe, (b) the mass enclosed within the horizon, and (c) the age of the Universe, when the temperature of the Universe was (i) 10^{16} GeV (a GUT phase transition), (ii) 100 GeV (the electroweak phase transition), (iii) 100 MeV (QCD phase transition), (iv) MeV (big-bang nucleosynthesis), and (v) an eV (decoupling).

5. In 1918, the observable Universe consisted of the Milky Way, which appeared to be in a steady state. Einstein modeled this as a closed FRW Universe with a nonzero cosmological constant. Show from the Friedmann equation that this allows a model with $H = 0$. Then show from the second Friedmann equation that $\ddot{a} \neq 0$ for this model. Thus, Einstein's model didn't work; it was static, but unstable.