Particle Astrophysics (171.697), Spring 2012
Problem Set 4
Due: first class of week 5

1. Consider a critical-density Universe in which massive neutrinos contribute Ω_ν to the density parameter. Show that on scales smaller than the neutrino Jeans length, perturbations in the remaining cold component grow as $\delta \propto t^\alpha$, where $\alpha = (\sqrt{25 - 24\Omega_\nu} - 1)/6$. (Hint: The Ω_ν of the critical density in neutrinos contributes to the expansion rate, but this component remains smoothly distributed.)

2. In this problem you will explore numerically the growth of a spherical perturbation in a cosmological-constant Universe. A spherically-symmetric perturbation collapses in a flat cosmological-constant Universe (i.e., $\Omega_m + \Omega_\Lambda = 1$) of arbitrary Ω_m. Derive an exact density contrast at virialization (you will probably not be able to do this analytically), and compare with the oft-quoted estimate, $1 + \delta = 178\Omega_m^{-0.7}$.

3. Consider linear growth of perturbations in a Universe with cold dark matter with density $\Omega_{\rm cdm} = 0.25$ and baryon density $\Omega_b = 0.05$. Consider only redshifts $z \gg 1$ so that the dynamical effect of the cosmological constant is negligible. Write down the differential equations for linear evolution of $\delta_{\rm cdm}(\vec{x}, t) = \delta\rho_{\rm cdm}(\vec{x}, t)/\bar{\rho}_{\rm cdm}$, the fractional perturbation to the CDM density, and for $\delta_b(\vec{x}, t) = \delta\rho_b(\vec{x}, t)/\bar{\rho}_b$, the fractional perturbation to the baryon density. Now consider the evolution of a single Fourier mode of wavelength λ and wavenumber $k = 2\pi/\lambda$, of the density field. Show that baryon perturbations are stabilized by pressure at small scales, and find an expression for the Jeans wavelength Λ_J, the wavelength that separates stable and unstable modes. Evaluate the Jeans wavelength just before and just after recombination, and determine the corresponding Jeans mass.