Particle Astrophysics (171.697), Spring 2012
Problem Set 5
Due: first class of week 6

1. Calculate the neutrino damping length as a function of the neutrino mass m_ν (for values of m_ν between 0.1 eV and 1 eV). Assume that all three neutrino species have the same mass.

2. Calculate the relation between the one-dimensional power spectrum $P_{1D}(k)$ (as would be measured in a "pencil-beam" survey) the usual three-dimensional power spectrum $P(k)$.

3. Calculate (numerically) and plot the root-variance $\sigma(M)$ as a function of mass M for the ΛCDM power spectrum $P(k)$ for $\Omega_m = 0.3$. Then, determine M_* (defined by $\sigma(M_*, t) = \delta_c$ as a function of redshift from $z = 0$ to $z = 100$. Use the proper linear-theory growth factor (you can use the semi-analytic approximation given in class), and also the proper δ_c (also using the approximation given in class).

4. Suppose that the 1-point distribution function, normalized to unit variance is $\mathcal{P}(\nu)$ (e.g., for Gaussian initial conditions, $\mathcal{P}\nu = (2\pi)^{-1/2}\exp(-\nu^2/2)$). (a) Show that the usual Press-Schecter equation for the number of gravitationally-bound halos with masses between M and $M + dM$ per comoving volume at redshift z generalizes to

$$\frac{dn}{dM}dM = \frac{f\rho_b}{M}\mathcal{P}[\nu(M,z)]\frac{\partial\nu(M,z)}{\partial M}dM,$$

where $\nu = \delta_c(z)/\sigma_M$, $\delta_c(z) = 1.69/D(z)$ is the critical overdensity for gravitational collapse, $D(z)$ is the linear-theory growth factor, and σ_M is the variance of the mass distribution for scales M. Also, $f = \int_0^\infty \mathcal{P}(\nu)\,d\nu$. (b) Now suppose that once halos form their mass is fixed, and suppose further that they disappear only when they merge into larger halos. Show that with these assumptions, the distribution (normalized to unity) of formation redshifts z_f for halos of mass M observed at redshift z_0 is

$$\frac{df}{dz_f} = \mathcal{P}'[\nu(M,z_f)]\frac{\partial\nu(M,z_f)}{\partial z_f}\left\{\mathcal{P}[\nu(M,z_0)]\right\}^{-1}.$$

(c) Evaluate this formation-redshift distribution for a Gaussian distribution of perturbations and describe it qualitatively. For hints, see *MNRAS* **321**, L7 (2001).