1. Show that for the fluid stress-energy tensor,

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu} + \Sigma^{\mu\nu},$$

   the stress tensor $\Sigma^{\mu\nu}$ can without loss of generality be taken to be traceless $\Sigma^{\mu}_{\mu} = 0$ and flow orthogonal $\Sigma^{\mu}_{\nu}u^{\nu} = 0$.

2. Show that Eqs. (4.32)-(4.34) in Bertschinger's article (astro-ph/9503125) are equivalent to Eq. (4.35)-(4.37). You can take $K = 0$ to simplify.

3. Consider a flat FRW Universe with nothing but nonrelativistic matter with density ρ and pressure $p = 0$. Consider now small-amplitude perturbations to this FRW spacetime, and consider perturbations of small spatial size, $\lambda \ll H^{-1}$. Use some combination of the continuity and Euler equations (4.24-25 in Bertschinger) and the synchronous-gauge or Poisson-gauge (your choice) equations of motion (i.e., Einstein equations) to show that the amplitude of these small-amplitude small-wavelength perturbations grow in time as $\delta\rho/\bar{\rho} \propto a \propto t^{2/3}$ with the scale factor $a(t)$. You should only need the Einstein equations for the scalar potential h and ξ (in synchronous gauge) or *phi* and ψ (in Poisson gauge).

4. Use linear theory to show that if (i) the transverse-vector perturbation is at some initial time nonzero, (ii) if its source vanishes (i.e., $\Sigma_{\perp,ij} = 0$), and (iii), the Universe consists only of pressureless matter, then transverse-vector-perturbation amplitude ($w_{\perp,i}$ in Poisson gauge and h_i in synchronous gauge) decays with the expansion. Derive this result in both synchronous and in Poisson gauge. To do so, you will need the continuity and Euler equations, as well as the relevant Einstein equation.