Particle Astrophysics (171.697)
Spring 2012

Problem Set 9

Due: In class, first class of week 10

1. **Direct detection of inflationary gravitational waves.** This problem will require you to look back a few weeks to our discussion of inflationary gravitational waves. Using the formulas derived in class, calculate the rms gravitational-wave (GW) amplitude in the inflationary GW background in the LIGO/LISA frequency band (use google to track down these sensitivities. An order-of-magnitude calculation should be enough.

2. **Constancy of superhorizon curvature.** If there are only scalar perturbations to the FRW metric, and if there are no anisotropic stresses, then the metric can be written,
$$ds^2 = a^2(\tau)\left[-(1+2\varphi)d\tau^2 + (1-2\varphi)d\vec{x}^2\right], \tag{1}$$
where $\varphi(t,\vec{x})$ is the scalar metric perturbation variable. (a) Show that the spatial curvature (i.e., the Ricci scalar for the three spatial dimensions) is
$${}^{(3)}R = \frac{4}{a^2}\nabla^2\varphi. \tag{2}$$
(b) Using the relevant Einstein, continuity, and Euler-Lagrange equations, show that the curvature perturbation $\mathcal{R}$ remains constant when a given Fourier mode is well outside the horizon. Dodelson's book and/or Bertschinger's Cosmological Dynamics article may be particularly useful for this problem. In fact, Dodelson more or less works it out, although in a different notation. If you can't at first work the problem out for yourself (it's not easy), then it may still be a good exercise to go through Dodelson's derivation and translate it into this notation and gauge choice.

3. **Cosmic variance:** Suppose the CMB temperature power spectrum $C_l^{\rm TT}$ is measured to the cosmic-variance limit for $l \leq 30$ to be consistent with $l(l+1)C_l = A$, where A is a power-spectrum amplitude derived from this CMB data. (a) Determine the cosmic-variance error with which this amplitude is determined. (b) Suppose further that an independent measurement (e.g., from galaxy surveys and/or CMB fluctuations on smaller scales) fixes the contribution A_s to this amplitude from density perturbations to be precisely the value that is measured from these ≤ 30. Now suppose that some additional contribution A_t (e.g., from gravitational waves) to this amplitude is postulated. What is the 3σ upper limit on A_t that can be derived from the data.

4. Suppose that there are some larger number N of sources of some fixed luminosity L randomly but uniformly distributed in space out to some distance R from us (these are distributed in Euclidean space; e.g., they might be sources in the Galactic halo). Suppose now that we measure the observed intensity $I(\hat{n})$ of these sources as a function of position $\hat{n}$ on the sky. Calculate the angular power spectrum C_l of this observed intensity map. If you're curious, you can also see how the power spectrum would change if the sources had a $1/R^2$ distribution in distance.