Ay101

Fall 2002

PHYSICS OF STARS

Problem Set 1

Due Wed, October 9, 2002

1. Assume a star obeys a linear density model so that

$$\rho(r) = \rho_c(1 - r/R),$$

where ρ_c is the central density and R is the radius of the star.

a. Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass (expressed in units of $M_\odot$).

b. Evaluate the central pressure P_c; i.e., find the values of A, x, and y in the expression $P_c = A(M/M_\odot)^x(R/R_\odot)^y$.

2. An eclipsing-binary system has a parallax of 0.1 arcsec, and for the moment we assume that this measurement is extremely accurate. It consists of two solar-mass stars identical to the Sun with a semi-major axis of 500 $R_\odot$. The period is very accurately known.

a. What is the angular size of each of the stars and of the semi-major axis? If you can measure angles on the sky with a 1σ rms accuracy of 0.01 arcsec, what is the percentage accuracy of the measurement of the semi-major axis and of the radius of each star?

b. Assume that the flux as a function of wavelength is given by a Planck function (i.e., its a blackbody) with effective temperature $T_{\rm eff} = 5800$ K. Assume that we have measurements of the flux ratio between $\log(\nu) = 14.0$ and 15.0 (where ν is given in Hz) that have an accuracy of 10%. With what precision can we determine the $T_{\rm eff}$ of the stars from these measurements?

c. What is the uncertainty in the mass of the system if the uncertainty in the parallax is taken as given in part a, 0.01 arcsec?

d. What is the luminosity of each star calculated from the $T_{\rm eff}$ and the R, and what is the uncertainty in the calculated luminosity of each star in this binary?

e. We can measure the apparent flux at the Earth from this star to an accuracy of 5%. Can we derive a more accurate luminosity and/or a more accurate radius from this observation than from relying on the measured parallax and the $T_{\rm eff}$ determined in part d?

3. Assume that the star-formation rate per unit mass of gas cloud is equal to 0 for $M \leq 0.05\, M_\odot$, and equal to KM^{-x}, where M is the stellar mass, K is a constant, and $x = 2.5$ for $M \geq 0.05\, M_\odot$. Assume that the star-formation rate has been constant for the age of the galaxy, which we take as 12 billion years. In addition, assume that the main-sequence lifetime varies as $K_1 M^{-3}$ for $M \leq 10\, M_\odot$ and $K_2 M^{-2}$ for $M \geq 10\, M_\odot$. The relationship between K_1 and K_2 is determined by the condition that the main-sequence lifetime is continuous at $M = 10\, M_\odot$.
   a. Determine the stellar mass M_* whose main-sequence lifetime is 12 Gyr.
   b. Let N_1, N_2, N_3, and N_4 equal the number of stars per unit volume in the mass intervals $M < M_*$, $M_* < M < 1.4\, M_\odot$, $1.4\, M_\odot < M < 10\, M_\odot$, and $M > 10\, M_\odot$. Find the predicted ratios N_2/N_1, N_3/N_1, and N_4/N_1.