Ay101

Fall 2002

PHYSICS OF STARS

Problem Set 2

Due Mon, October 14, 2002

1.
   a. Calculate approximately for the Sun the gravitational potential energy, the thermal energy, the rotational energy (assume solid-body rotation (assume solid-body rotation with an angular velocity of 4.3×10^{-7} sec^{-1}), the kinetic energy of bulk flows in the surface convective zone (5% of the total mass of the Sun, moving at 1.5 km/sec), and the energy in magnetic fields (assume the average surface magnetic field of 1 Gauss is uniform throughout the Sun).
   b. Calculate the ratio of the gas pressure to the radiation pressure at the center of the Sun.
   c. From your results, do you think that we can safely ignore rotation and magnetic fields when discussing stellar structure for stars like the Sun?

2. Evaluate the expression for the adiabatic gradient, $\nabla_{\rm ad} = (\partial \ln T/\partial \ln P)_{\rm ad}$ for a pure hydrogen gas,

$$\nabla_{\rm ad} = \frac{2 + 2\mathcal{D}(3/2 + \chi_H/kT)}{5 + w\mathcal{D}[\chi_h/kT + (3/2 + \chi_H/kT)(5/2 + \chi_H/kT)]},$$

   where $\mathcal{D}(y) = y(1-y)/[(2-y)(1+y)]$, y is the ionization fraction, and χ_H is the ionization potential of hydrogen. Do this for $T = 3,000$, 10,000, and 30,000 K. Evaluate $\nabla_{\rm ad}$ as a function of y for $y = 0$ to 1 in steps of 0.1. Comment on the maximum and minimum value of $\nabla_{\rm ad}$ that is achieved over this range.

3.
   a. The amount of energy we receive from the Sun per cm^2 per second just above the atmosphere of the Earth is the solar constant. It has a value of $S = \pi f(\text{Sun}) = 1.38 \times 10^6$ erg/cm^2/sec. Using the distance of the Earth to the Sun, what is the surface flux πF on the Sun. What effective temperature does this imply for the Sun?
   b. The mean temperature of the Earth is about 300 K. What fraction of the incident solar energy goes to heat the Earth and what fraction is reflected back into space? (What approximations, if any, have you made to derive an answer?)

c. What fractional change in $T_{\rm eff}$ for the Sun would change the mean temperature of the Earth by 10 K?

4. A gas in thermodynamic equilibrium consists only of hydrogen with a number density of 10^{23} atoms/cm^3.
   a. Calculate the ratio of neutral to ionized hydrogen as a function of temperature T for T between 3,000 and 50,000 K. Plot your results.
   b. For the same gas with the same number density and range of T, calculate the number densities in the first 10 levels of neutral hydrogen, $n = 1$ to 10, and plot your results for each n as a function of T.