Ay101

Fall 2002

PHYSICS OF STARS

Problem Set 3

Due Mon, October 21, 2002

1. Assume the specific intensity of the radiation field is $I = I_0 + I_1\mu$, where $\mu = \cos\theta$. Solve for the radiation pressure $P_{\rm rad}$ and the energy flux carried by radiation in terms of I_0 and I_1. Show that in the solar interior, where all the luminosity is generated at $r < 3 \times 10^{10}$ cm and $T > 3 \times 10^6$ K, I_1 is much less than I_0.

2. A certain gas emits radiation thermally at the rate of $P(\lambda)$ (power per unit volume and wavelength range). A spherical cloud of this gas has radius R and temperature T and is a distance d from the Earth ($d \gg R$).
   a. Assume that the cloud is optically *thin*. What is the brightness of the cloud as measured on Earth? Give your answer as a function of the impact parameter b, the point of closest approach of the light ray to the center of the cloud (i.e., an impact parameter $b = 0$ describes a ray that passes through the center of the cloud and $b = R$ corresponds to a light ray from the edge of the cloud).
   b. What is the effective temperature of the cloud?
   c. What is the flux πF_λ measured at Earth coming from the entire cloud?
   d. Anwer parts (a)–(c) for an optically *thick* cloud.

3. Consider a plane-parallel atmosphere that consists of two parts, a semi-infinite lower part at temperature T_l that lies below an upper part at temperature T_u, and assume a source function $S_\lambda = B_\lambda(T)$. The surface of the atmosphere (i.e., optical depth $\tau = 0$) is at the top of the upper part, and the optical depth of the upper part is τ_u. Solve the radiative transfer equation to determine the emergent intensity $I(\tau = 0, \theta)$ as a function of viewing angle θ.

4. The center to limb variation for the solar radiation at 501 nm has been measured to be

$$\frac{I_\lambda(0,\theta)}{I_\lambda(0,0)} = 0.2593 + 0.8724\cos\theta - 0.1336\cos^2\theta.$$

   The central value is $I_\lambda(0,0) = 4.05 \times 10^{14}$ erg cm^{-2} sec^{-1} cm^{-1}. Calculate $S_\lambda(\tau_\lambda)$. Assume $S_\lambda = B_\lambda(T(\tau_\lambda))$. Calculate $T(\tau_\lambda)$, where τ_λ is the optical depth at 501 nm. Plot $S_\lambda(\tau_\lambda)$ and $T_\lambda(\tau_\lambda)$.