Ay121

Fall 2007

RADIATIVE PROCESSES

Final Exam

Please hand in to me in my office (Bridge Annex 120) before 5pm on Tuesday Dec 18, or to Shirley Hampton if I'm not in.

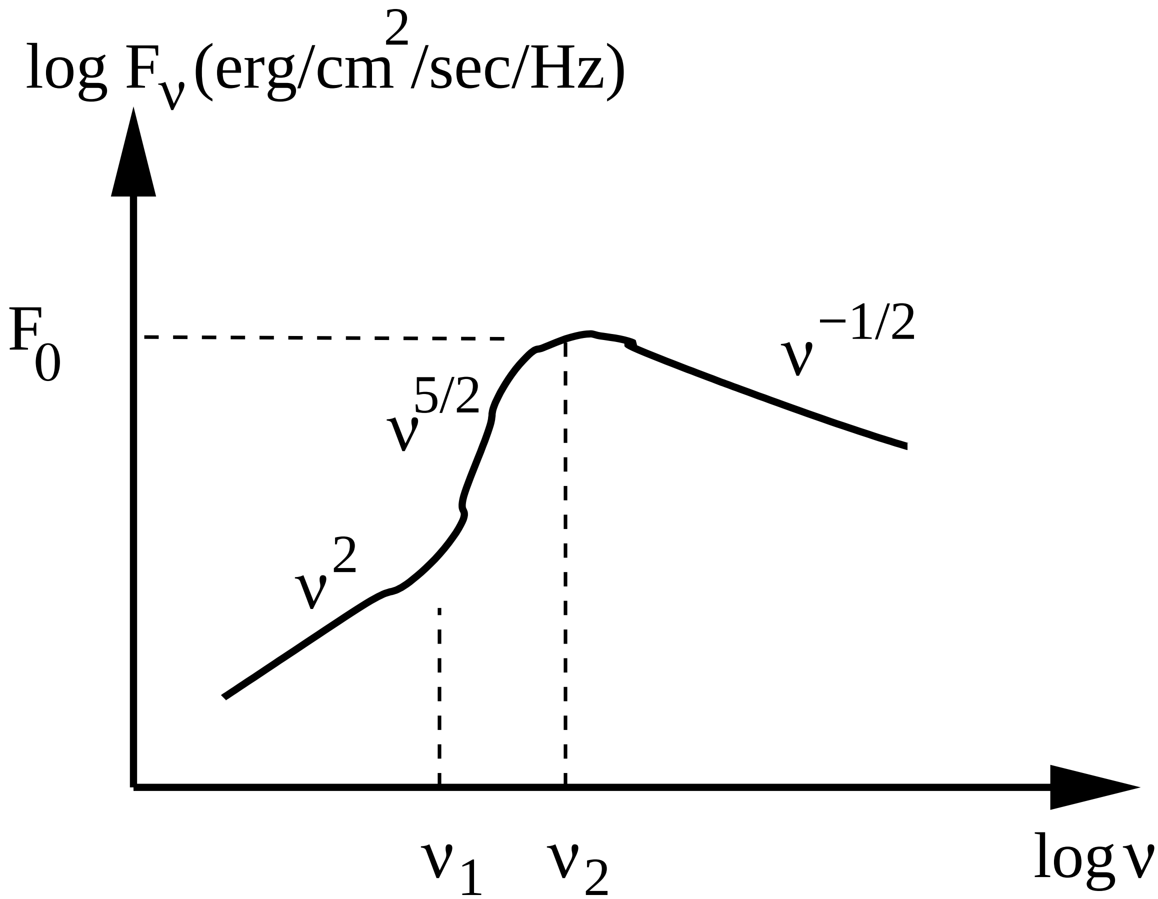
Instructions: Set aside three contiguous hours and go somewhere quiet where you will not be disturbed (e.g., an abandoned classroom or a remote corner of a remote library). Solve the problems below to the best of your ability. Please do **not** refer to any books or notes. If there is a solution that requires some numerical constant you do not remember nor have access to (note the list of constants provided at the end of the test), or particular formula whose detailed form you do not remember, then specify precisely what you would need to solve the problem. Please write your solutions as clearly as possible and put a box around the final answer.

1. A supernova remnant has an angular diameter $\theta = 4.3$ arcmin and a flux at 100 MHz of $F_{100} = 1.6 \times 10^{-19}$ erg cm^{-2} sec^{-1} Hz^{-1}. Assume that the emission is thermal.
   a. What is the brightness temperature T_b? What energy regime of the blackbody curve does this correspond to?
   b. The emitting region is actually more compact than indicated by the observed angular diameter. What effect does this have on the value T_b?
   c. At what frequency will this object's radiation be maximum if the emission is blackbody?
   d. What can you say about the temperature of the material from the above results?

2. An optically thin cloud surrounding a luminous object is estimated to be 1 pc in radius and to consist of ionized plasma. Assume that electron scattering is the only important extinction mechanism and that the luminous object emits unpolarized radiation.
   a. If the cloud is unresolved (angular size smaller than angular resolution of detector), what is the net polarization observed?
   b. If the cloud is resolved, what is the polarization direction of the observed radiation as a function of position on the sky? Assume that only single scattering occurs.
   c. If the central object is clearly seen, what is an upper bound for the electron density of the cloud, assuming that the cloud is homogeneous?

3. It is frequently argued that a source of radiation which undergoes a fluctuation of duration Δt must have a physical diameter of order $D \lesssim c\Delta t$. Suppose, however, that the

source is an optically thick spherical shell of radius $R(t)$ that is expanding with relativistic velocity $\beta \sim 1$ and Lorentz factor $\gamma \gg 1$. Derive an upper limit to the source radius if an observer sees a fluctuation of duration Δt at time t, and show that it is much larger than $c\Delta t$.



4. The spectrum shown in the Figure is observed from a point source of unknown distance d. A model for this source is a spherical mass of radius R that is emitting synchrotron radiation in a magnetic field of strength B. The space between us and the source is uniformly filled with a thermal bath of hydrogen that emits and absorbs mainly by bound-free transitions, and it is believed that the hydrogen bath is unimportant compared to the synchrotron source at frequencies where the former is optically thin. The synchrotron source function can be written as

$$S_\nu = A(\text{erg cm}^{-2}\text{ sec}^{-1}\text{ Hz}^{-1})(B/B_0)^{-1/2}(\nu/\nu_0)^{5/2}.$$

The absorption coefficient for synchrotron radiation is

$$\alpha_\nu^s = C(\text{cm}^{-1})(B/B_0)^{(p+2)/2}(\nu/\nu_0)^{-(p+4)/2},$$

and that for the bound-free transition is

$$\alpha_\nu^{bf} = D(\mathrm{cm}^{-1})(\nu/\nu_0)^{-3},$$

where A, B, C, and D are constants and p is the power-law index for the assumed power-law distribution of relativistic electrons in the synchrotron source.

a. Find the size of the source R and the magnetic field strength B in terms of the solid angle $\Omega = \pi(R/d)^2$ subtended by the source and the constants A, B_0, ν_0, C, and D.

b. Now using D and ν_1, in addition to the previous constants, find the solid angle of the source and its distance.

5. Give the spectroscopic terms arising from the $3p4p$ configuration, using L-S coupling. Include parity and J values. Evaluate the degeneracy of the $3p4p$ configuration from the l values involved. Next evaluate the degeneracy from the L and S values. Finally, evaluate the degeneracy of each of the levels from the J values. Show that these degeneracies are consistent, in that the degeneracy of the configuration is equal to the sum of the degeneracies of the terms it generates, and that the degeneracy of any term is equal to the sum of the degeneracies of the levels it generates.

6. Which of the following transitions are allowed under L-S coupling selection rules for dipole transitions and which are not? Explain which rules, if any, are violated.

a. $3s\ {}^2S_{1/2} \leftrightarrow 4s\ {}^2S_{1/2}$

b. $2p\ {}^2P_{1/2} \leftrightarrow 3d\ {}^2D_{5/2}$

c. $3s3p\ {}^3P_1 \leftrightarrow 3p^2\ {}^1D_2$

d. $2p3p\ {}^3D_1 \leftrightarrow 3p4d\ {}^3F_2$

e. $2p^2\ {}^3P_0 \leftrightarrow 2p3s\ {}^3P_0$

f. $3s2p\ {}^1P_1 \leftrightarrow 2p3p\ {}^1P_1$

g. $2s3p\ {}^3P_0 \leftrightarrow 3p4d\ {}^3P_1$

h. $1s^2\ {}^1S_0 \leftrightarrow 2s2p\ {}^1P_1$

i. $2p3p\ {}^3S_1 \leftrightarrow 2p4d\ {}^3D_2$

j. $2p^3\ {}^2D_{3/2} \leftrightarrow 2p^3\ {}^2D_{1/2}$

Possibly useful constants:

speed of light: $c = 3.0 \times 10^{10}$ cm/sec

Planck's constant: $h = 6.63 \times 10^{-27}$ erg-sec

electron charge magnitude: $e = 4.8 \times 10^{10}$ esu

electron mass: $m_e = 9.11 \times 10^{-28}$ g

proton mass: $m_p = 1.67 \times 10^{-24}$ g

fine-structure constant: $\alpha = e^2/\hbar c \simeq 1/137$

classical electron radius: $r_e = e^2/m_e c^2 = 2.82 \times 10^{-13}$ cm

electron Compton wavelength: $\lambda_e = \hbar/m_e c = r_e/\alpha = 3.86 \times 10^{-11}$ cm

Rydberg energy: $m_e e^4/2\hbar^2 = m_e c^2 \alpha^2/2 = 13.6$ eV

Thomson cross section: $\sigma_T = 8\pi r_e^2/3 = 6.65 \times 10^{-25}$ cm^2

Newton's constant: $G_N = 6.67 \times 10^{-8}$ cm^3/g/sec^2

Boltzmann constant: $k = 1.38 \times 10^{-16}$ erg/K$= 8.62 \times 10^{-5}$ eV/K

Stefan-Boltzmann constant: $\sigma = \pi^2 k^4/60\hbar^3 c^2 = 5.67 \times 10^{-5}$ erg/cm^2/deg^4/sec

parsec: pc$=3.09 \times 10^{18}$ cm

solar mass: $M_\odot = 1.99 \times 10^{33}$ g

Planck function: $B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1}$