Ay121

Fall 2007

RADIATIVE PROCESSES

Problem Set 2

Due in class October 18, 2007

1. Molecules of a particular type are mixed with H_2 molecules (which are the dominant species, of density n and kinetic temperature T_K) in the microwave background, with radiation temperature $T_R = 2.7$ K. The molecules of interest have a transition frequency ν between a stable (or metastable) lower state l and an upper state u. Th upper state can be excited radiatively, as discussed in class and in RL. However, this upper state can also be excited via collisions with H_2 molecules. The rate at which collisions with H_2 molecules excite and de-excite the u level is described by a collisional de-excitation rate coefficient σv (units of cm^3/sec). Although we have not defined this coefficient precisely, you should be able to determine its precise meaning by working through this problem.

   a. By solving the rate equations, show that in equilibrium, the excitation temperature T_S of the molecular levels is given by

   $$\exp(-h\nu/kT_s) \equiv \frac{n_u g_l}{n_l g_u} = \frac{\exp(-h\nu/kT_R) + \xi \exp(-h\nu/kT_K)}{1+\xi},$$

   where

   $$\xi = \frac{n\sigma v}{A_{ul}}[1 - \exp(-h\nu/kT_R)],$$

   and A_{ul} is the Einstein A coefficient for the ul transition. (HINT: First, imagine that the transition could proceed only collisionally, and *not* radiatively, and use the balance between collisional excitation and de-excitation in thermal equilibrium to figure out what the collisional rate coefficient is. Then include radiative transitions to get the result above. Your solution should involve some algebra, but *no* integrals.)

   b. Show that at low densities, $T_s = T_R$, and that at high densities $T_s = T_K$.

   c. Show that if $kT_K \gg h\nu$, the excitation temperature will be twice the background temperature if

   $$n = \frac{A_{ul}}{\sigma v}\frac{\exp(-h\nu/2kT_R)}{1 - \exp(-h\nu/kT_R)}.$$

   d. The collisional excitation cross section for most large molecules by H_2 is about $(3\mathring{A})^2$. For $T_K = 100$ K, compute for each of the following molecules the critical density above which the excitation temperature will rise above twice the background temperature (cf. part c).

	ν(GHz)	$A_{ul}(\mathrm{sec}^{-1})$
OH	1.667	8×10^{-11}
NH_3	23.69	4×10^{-8}
HCN	88.63	2×10^{-5}

2. In this problem we consider the escape of photons in an optically thick Doppler-broadened line. The cross section for scattering in the line is given by

$$\sigma(\nu) = \frac{h\nu}{4\pi} B\phi(\nu),$$

where the Doppler profile function is

$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}} \exp[-(\nu - \nu_0)^2/(\Delta\nu_D)^2].$$

Suppose that there is no correlation between the incoming and scattered photons and that their frequencies are independently distributed over the line profile $\phi(\nu)$. Consider photons released in the middle of a spherical cloud of radius R and optical depth at the line center $\tau_0 \equiv \tau(\nu_0) \gg 1$ between the center and surface of the sphere.

a. Prove that the mean number of scatterings before escape is $N \simeq \tau_0(\ln \tau_0)^{1/2}$.

b. Contrast this form of escape with that of photons trapped in a medium with coherent and energy-independent scattering.

c. Estimate the specific flux of energy radiated from the surface of a uniform spherical cloud of radius R within which the transition forming the line has excitation temperature T_e. Do this by calculating the net rate at which photons created in spontaneous decays escape from the cloud. Assume that τ_0 is the line-center optical depth to the center of the cloud. For simplicity, treat the factor $(\ln \tau_0)^{1/2}$ as being of order unity.

d. Provide a physical interpretation for the result you obtain in c.

3. In this problem, we model flat dusty disks aroung young stars. Substantial infrared (IR) emission has been detected aroung young "T Tauri" stars that are still being born. The IR excess is thought to originate from a dusty accretion disk surrounding the young star that absorbs visible light and re-radiates at IR wavelengths. Recently, objects have been discovered by HST in the Orion Nebula dubbed "proplyds": objects of roughly solar-system size (50–1000 AU) whose silhouettes are seen against the bright background of the emission nebula. Assume that the mass density of dust grains is 3 g cm^{-3}.

a. Consider a flat dusty disk whose surface mass density falls as

$$\Sigma \simeq 10^4 a_{\mathrm{AU}}^{-3/2} \text{ g cm}^{-2},$$

where $a_{\rm AU}$ is the disk radius measured in AU. Take the dust to consist of grains ~ 1 micron in size, so that they present a roughly geometrical extinction cross section to radiation of visible wavelengths. Take the mass density of dust to be 1% of the total mass density of the disk. At what radius does the optical depth to visible light reach unity? (Note: it is the optical depth in the VERTICAL direction that we seek. That is, imagine viewing the disk face-on, so that we see a pancake in the sky. At some radius of the disk, the optical depth integrated along the line of sight—the "vertical" direction, perpendicular to the disk's radial direction—will reach unity). Is it comparable to the size of proplyds?

b. Assuming the disk is (1) entirely flat, and (2) radiates like a blackbody, calculate the temperature of the disk as a function of radius a for $a \gg R_*$, the stellar radius. Take the star to be a blackbody as well, of effective temperature T_*. Assume the disk is heated entirely by radiation from the star, and make clear whatever other approximations you are using.