Ay121

Fall 2007

RADIATIVE PROCESSES

Problem Set 3

Due in class October 25, 2007

1. In this problem we apply the Eddington approximation with boundary conditions from the two-stream approximation to determine what happens when an incident radiative flux F_I falls on a planetary atmosphere that only scatters radiation (no absorption or emission) that lies above a ground that absorbs all radiation. Let F_R be the reflected flux. Take the atmosphere to have optical depth τ_* and the ground to be completely absorbing (i.e., neglect any energy emitted by the ground).
   a. Calculate the mean intensity $J(\tau)$ as a function of optical depth τ in the atmosphere.
   b. Solve for F_R/F_I.
   c. Determine the limb darkening function $I(\mu)/I(0)$.

2. Now we repeat the first problem, but this time we suppose that there is some absorption/emission, so that the single-scattering albedo is $0 < 1 - \epsilon < 1$. We also now assume that the atmosphere is infinitely deep.
   a. Calculate $J(\tau)$.
   b. Derive an expression for F_R/F_I.
   c. Calculate the limb darkening function.

3. Consider a plane parallel grey (i.e., emits and absorbs the same at all frequencies) atmosphere of optical depth τ_* that carries a constant flux F of radiation. Assume that the atmosphere is in local thermodynamic equilibrium. Integrate the equations of radiative transfer over frequency.
   a. Determine $T(\tau)$.
   b. Verify that $T(0)/T_e < 1$, where T_e is the effective temperature defined by $F = \sigma T_e^4$, and explain this result.
   c. Calculate the limb darkening function.
   d. Estimate τ_* for Venus in the thermal infrared. A few percent of the sunlight incident on the planet reaches the ground which has a $T_s \simeq 740$ K.

4. This problem is meant to help you review some theory of electromagnetic fields, hopefully in an interesting way. One of Maxwell's equations, $\vec{\nabla} \cdot \vec{B} = 0$, is a consequence (or statement) that there are no magnetic charges. However, suppose that magnetic charges

existed. Then this Maxwell equation would become $\vec{\nabla} \cdot \vec{B} = 4\pi\rho_m$, where ρ_m is the magnetic-charge density.

a. Suppose that there is an electric charge e at the origin and a magnetic monopole (a magnetic charge) g at position $\vec{x} = \vec{R}$. Show that the electromagnetic fields have a nonzero angular momentum and determine its magnitude and direction.
b. Now suppose that an electric charge e approaches a magnetic monopole (assumed to be much heavier than the electric charge and thus taken to be at rest) at an impact parameter b. Calculate the deflection angle of the electric charge as it passes the monopole.
c. Calculate the amount of energy radiated in the scattering process of part b.