Ay121

Fall 2007

RADIATIVE PROCESSES

Problem Set 4

Due in class November 1, 2007

1. An electromagnetic wave of frequency ω is incident on a conducting sphere of radius R. Calculate the cross section for scattering of the EM wave in the low-frequency limit. What is the maximum frequency for which your calculation works?

2. Take a good look at Eq. (3.60b) in Rybicki & Lightman. Determine whether this result comes from assuming that the radiative-reaction term is a third derivative in the harmonic-oscillator equation of motion or whether it is a first derivative. How would the cross section, Eq. (3.62), and its low- and high-frequency limits change if the other assumption (i.e., first or third derivative) was used in the derivation of (3.60b)?

3. A good fraction of the Milky Way's diffuse hard-γ-ray background is due to decays of π^0 mesons.
   a. Find the predicted γ-ray spectrum due to $\pi^0 \to 2\gamma$ decay for π^0's with velocity βc in the observer's frame (the pion has a mass $m_\pi = 135$ MeV).
   b. Most π^0's are produced in interstellar space when a cosmic-ray proton collides with an ambient atom via the reaction, $p + p \to p + p + \pi^0$. Assuming the target proton is at rest, calculate the threshold energy for the cosmic-ray proton for this process to occur.
   c. Since the cosmic-ray energy energy spectrum falls very steeply, most pions are produced at threshold. Calculate the γ-ray spectrum resulting from the decay of these threshold pions.

4. The Universe is filled with a homogeneous cosmic microwave background (CMB) with a blackbody spectrum at a temperature $T_{\rm cmb} = 2.7$ K.
   a. Suppose we are moving with a velocity $\mathbf{v}$ (we are). Calculate the observed intensity $I_\nu(\theta)$ as a function of the angle θ between the line of sight $\hat{\mathbf{n}}$ and $\mathbf{v}$. Give an exact expression. Do *not* assume that this velocity small compared with the speed of light (even though it is—370 km/sec).
   b. Show that to lowest order in v/c, the induced temperature pattern is a dipole.

c. Now consider the temperature pattern to quadratic order in v/c, and show that there is a temperature quadrupole moment and determine its amplitude. At what angle does this quadrupole produce a temperature maximum?

d. In addition to the CMB, there is also an unresolved diffuse extragalactic cosmic infrared background (CIB). It is believed that with sufficient angular resolution, we should be able to resolve this CIB into discrete sources. Again, assuming we are moving with a velocity $\mathbf{v}$, determine how the source density (number of sources per unit solid angle) should vary with angle θ. Also, estimate the source density required to detect this anisotropy.

e. Determine how the received flux from each object should vary with angle θ, and make sure that the answer to this and (d) are consistent with that of (a).