Ay121

Fall 2007

RADIATIVE PROCESSES

Problem Set 6

Due in class November 15, 2007

1. **The Sunyaev-Zeldovich Effect.** The Sunyaev-Zeldovich (SZ) effect results from the scattering of cosmic microwave background (CMB) photons by high-energy (but non-relativistic) electrons in clusters of galaxies. Thus, $\tilde{x} \equiv h\nu/kT_e \ll 1$, where T_e is the electron temperature, and ν is the photon frequency. The Compton-y parameter for this case is

$$y_{\mathrm{SZ}} \equiv \int_{t_0}^{t} \frac{kT_e}{m_e c^2} n_e \sigma_T c \, dt.$$

a. Show that the Kompaneets equation in this case may be written

$$\frac{\partial n}{\partial y_{\mathrm{SZ}}} = \frac{1}{\tilde{x}^2} \frac{\partial}{\partial \tilde{x}} \left[\tilde{x}^4 \left(n + n^2 + (\partial n / \partial \tilde{x}) \right) \right].$$

b. Show that this implies that

$$\frac{\partial n}{\partial y_{\mathrm{SZ}}} \simeq \frac{1}{\tilde{x}^2} \frac{\partial}{\partial \tilde{x}} \left[\tilde{x}^4 \frac{\partial n}{\partial \tilde{x}} \right],$$

and hence that

$$\frac{\partial n}{\partial y_{\mathrm{SZ}}} \simeq \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \frac{\partial n}{\partial x} \right],$$

where $x = h\nu/kT_{\mathrm{rad}}$, and T_{rad} is the radiation temperature.

c. Write down the occupation number for photons in blackbody radiation and hence show that

$$\frac{\partial n}{\partial y_{\mathrm{SZ}}} = \frac{x e^x}{(e^x - 1)^2} \left[x \frac{e^x + 1}{e^x - 1} - 4 \right].$$

d. In the SZ effect we have small y factors (and small optical depths) so the previous equation may be written as a difference equation with $y = 0$ at $t_0 = 0$. Hence show that

$$\frac{\Delta n}{n} = \frac{\Delta I_\nu}{I_\nu} = \frac{x e^x}{(e^x - 1)^2} y_{\mathrm{SZ}} \left[x \frac{e^x + 1}{e^x - 1} - 4 \right].$$

e. Use the previous results to show that

$$\frac{\Delta T_{\rm rad}}{T_{\rm rad}} = y_{\rm SZ}\left[x\frac{e^x+1}{e^x-1} - 4\right],$$

and hence that at low frequencies,

$$\frac{\Delta T_{\rm rad}}{T_{\rm rad}} \simeq -2y_{\rm SZ}.$$

f. Estimate $(\Delta T_{\rm rad}/T_{\rm rad})$ in terms of $y_{\rm SZ}$ at 32 GHz assuming $T_{\rm rad} = 2.726$ K.

2. **High-redshift radio galaxies.**

a. The first high-redshift ($z > 3$) galaxies discovered were radio galaxies with unusually steep radio spectra. Suppose that relativistic electrons are injected at a constant rate into the hot spot of a double radio source with a power-law spectrum, $S(\gamma)d\gamma \propto \gamma^{-p}d\gamma$, and reside there for a fixed time t_r before being advected and expanded away by the backflow. Suppose that the dominant energy loss of the electrons is through inverse Compton scattering of CMB photons. Argue that the energy density in CMB photons scales as $U_{\rm cmb} \propto (1+z)^4$. Show that the steady-state spectrum of relativistic electrons breaks from γ^{-p} to $\gamma^{-(p+1)}$ at $\gamma \sim m_e c^2/(\sigma_T c U_{\rm cmb} t_r)$. Thus, show that the radio spectrum of synchrotron radation from these electrons steepens from $I_\nu \propto \nu^{-(p-1)/2}$ to $I_\nu \propto \nu^{-p/2}$ at a break frequency ν_b which scales with redshift as $\nu_b \propto (1+z)^{-9}$. Could this explain why radio sources with steep spectra turn out to be at high redshift, while similar ones with flat spectra turn out to have low redshifts ($z < 1$)?

b. Another remarkable property of these radio sources ist that the optical emission from these galaxies is observed to be aligned with the radio lobes. One explanation is that the optical emission is light from a central quasar that escapes from a surrounding dust torus only along an axis punched through the dust by the radio jet. The polarization of this emission in some sources favors this proposal. Some authors have argued that the neutral colors of the optical emission favor Thomson scattering by ionized gas falling into the galaxy over dust scattering as the source. In the $z = 3.8$ source 0647+415, the radio and optical emission has a linear extent of 10 kpc, and the extended optical emission has a luminosity of 4×10^{44} erg s^{-1}. Given that the brightest unobscured quasars have optical luminosities of 10^{47} erg s^{-1}, if such a quasar were at the center of this source, what would be the mass in ionized gas required to scatter the extended optical emission?

3. In the last problem set, you used the observed synchrotron radiation from the Crab nebula to estimate its magnetic field and the population of relativistic electrons in the nebula. The same relativistic electrons that produce the synchrotron radiation will also inverse-Compton scatter ambient photons.

a. Use the figure from the last problem set to estimate the energy density in synchrotron photons in the Crab. Also estimate the energy density in CMB photons.
b. What is the ratio of inverse-Compton cooling rate to the synchrotron cooling rate for a relativistic electron in the Crab nebula?
c. In this part, neglect the inverse Compton scattering of synchrotron photons and consider only the inverse Compton scattering of CMB photons. Given the synchrotron spectrum in the figure of the last problem set and assuming a mean magnetic field in the nebula equal to the equipartition value $B = 2 \times 10^{-4}$ G, compute the spectrum of inverse Compton scattered gamma rays (approximate the CMB-photon energy distribution and the scattering functions as delta functions). Sketch the result on an extension of the figure of the last Problem set to $> 10^{27}$ Hz (> 4 TeV). How does the amplitude of the gamma-ray flux vary with the assumed B?
d. The gamma-ray spectrum observed at Earth from the Crab nebula is, as a function of photon energy $E = h\nu$,

$$J(E) = 3.2 \times 10^{-7} (E/1\,\mathrm{TeV})^{-2.5}\,\mathrm{photons\,m^{-2}\,sec^{-1}\,TeV^{-1}},$$

for 0.5 TeV$< E <$ 10 TeV (photons of these energies are individually detected looking for flashes of atmospheric Cerenkov radiation from the air showers created when the photons strike the Earth's atmosphere). Under the assumption of part c, compute B and compare to the results that you got in the last problem set.
e. How will the inclusion of inverse Compton scattering of synchrotron photons ("synchrotron self-Compton scattering") modify your results of parts c and d? Be at least somewhat quantitative, and include sketches like the one in part c.