Ay121

Fall 2007

RADIATIVE PROCESSES

Problem Set 8

Due Tuesday, December 11, 2007

1. This problem is a followup to problem 1 of Problem Set 2.
   a. HCN is a linear molecule with a tiny moment of inertia about the long axis and a moment of inertia $I = 11.404$ amu $\AA^2$ about both short axes. What is the frequency of the $J = 1$ to $J = 0$ rotational transition?
   b. HCN has a dipole moment of 2.984 Debye (1 Debye= 10^{-18} esu-cm). Use this to estimate the Einstein A coefficient for the $J = 1$ to $J = 0$ rotational transition.
   c. OH is also a linear molecule with moment of inertia $I = 0.909$ amu $\AA^2$. What is the frequency of the $J = 1$ to $J = 0$ rotational transition? What do you think is the nature of the 1.667 GHz transition listed in problem 1 of Problem Set 2?
   d. The ammonia transition in the table of problem 1 of Problem Set 2 is the oscillation of the nitrogen through the plane of the hydrogen atoms. Using your knowledge of chemical bonds and radiative processes, derive as best you can the numerical value of the A_{ul} coefficient.

2. In order for the first stars in the Universe to form, primordial metal-free gas must be able to cool to low temperatures. Since molecular hydrogen has low-lying rotational levels, it is believed that it may play an important role in cooling gas at temperatures below 10^4 K, where atomic-hydrogen cooling becomes ineffective. Here we consider this problem and also consider the possible role of cooling by HD molecules.
   a. Argue that atomic-hydrogen cooling becomes ineffective below temperatures 10^4 K.
   b. H_2 is a homonuclear molecule, so radiative transitions within the lowest electronic state involve only quadrupole transitions $\Delta J = 2$. Show that the energy of the transition is given by

   $$E_{J+2,J} = 2kB_r(2J + 3) = kT_{J+2,J},$$

   and estimate the rotational constant B_r (which is experimentally known to be $B_r =$ 85.33 K) and thus the transition temperature $T_{2,0}$ for transitions to the ground state.
   c. Since the centers of mass and charge do not coincide in HD, it may undergo dipole transitions between adjacent rotational levels. Show that the $J + 1 \to J$ transition energy is

   $$E_{J+1,J} = 2kB_r(J + 1) = kT_{J+1,J},$$

and estimate the rotational constant B_r (which is experimentally known to be $B_r =$ 64.3 K) and thus the transition temperature $T_{1,0}$ for transitions to the ground state.

d. Show that the Einstein A coefficient for spontaneous decays from $J = 1$ to $J = 0$ in HD is

$$A_{1,0}^{\mathrm{HD}} = \frac{512\pi^4 k^3}{3h^4c^3} \frac{(B_r^{\mathrm{HD}})^3 \mu_{\mathrm{HD}}^2}{3},$$

and evaluate it numerically (using the experimental value for B_r^{HD}).

e. Now consider the $J = 2 \to J = 0$ transition for H_2, a quadrupole transition. Estimate roughly the A coefficient (the exact result is 2.44×10^{-11} sec).

f. Now suppose that the primordial abundances of deuterium and hydrogen are $n_D/n_H = 2 \times 10^{-5}$. At what temperatures (if any) does HD cooling become more important than H_2 cooling? Assume that equilibrium abundances of all species are maintained by frequent collisions.

f. You should have found in part e that HD cooling becomes important at temperatures $T \lesssim 200$ K. Show that our neglect of higher rotational states in our analysis is justified at these temperatures.