Cataclysmic Variables:

Are binaries with WD and companion; "Typical" CV has sub-stellar-mass MS star as companion but some may also have giant star (e.g., in recurrent nova T Coronae Borealis) or WD companion (e.g., short period — $P \simeq 0.3$ hr — AM Canum Venaticorum).

Kepler's law: $a = (3.5\times10^{10}\ \text{cm})(P/\text{hr})^{2/3} M^{1/3}$

($R_\odot = 7\times10^{10}$ cm $R_{WD} \simeq 5\times10^{8}$ cm).

Radiation may be seen from ① Secondary star, ② accretion disk, ③ bright spot (where accretion stream hits disk); and ④ WD surface + boundary layer between WD and disk.

In magnetic variables (e.g. AM Herculis), accreting matter may be channelled by WD B field directly to WD poles, w/o disk.

"Variable" because have periods 2-10 hr (except for WD companion, $P \sim 0.3$ hr, or giant companions, $P \sim 200$ days).

"Cataclysmic" because undergo outbursts:

*Classical nova: thermonuclear detonation of ~~[illegible]~~ H accreted on WD $\Rightarrow$ expected to recur $\sim 10^4$ yr
very bright

Dwarf nova: sudden increase in brightness of accretion disk. $\Longleftrightarrow$ frequent small outbursts

Some CVs (e.g., magnetic AM Her and VY Scl) may have "high" and "low" states.

Recurrent novae: outbursts (smaller than classical novae) ~~[illegible]~~with 10-50 yr intervals

Energetics:

The luminosity is $L = \frac{G M_{WD} \dot{M}}{R_{WD}}$; half emitted in disk, and other half in boundary layer where disk meets star:

$$L_{disk} = L_{BL} \simeq \frac{G M_{WD} \dot{M}}{2 R_{WD}}$$

e.g., $M_{WD} = M_\odot$ $R_{WD} = 6\times10^8$ cm $\dot{M} = 1.6\times10^{-10} M_\odot/\text{yr}$, $L \simeq 10^{33}$ erg/sec

If all ~~energy~~ disk luminosity came from outer edge of disk, then, would have

$$(\overset{\text{2 sides of disk}}{2}\pi R_d^2)(\sigma T_{out}^4) \simeq \frac{G M_{WD} \dot{M}}{2 R_d}$$

e.g., $R_d = 4\times10^{10}$ cm $\Rightarrow T_{out} \simeq 2\times10^3$ K.

If all comes from inner edge, $R_d \to R_{WD}$, and

$$T_{max} \simeq 5\times10^4 \text{ K}.$$

So disk radiates at $T \simeq 10^3 - 10^5$ K

$\Rightarrow$ visible (3000–10,000 Å) — UV (1000–3000 Å)

Boundary layer typically occupies smaller area, and so ~~then~~ inferred temperature is higher ~3×

$\Rightarrow$ BL radiates in EUV (300–1000 Å) or soft X-ray (50–300 Å)

May also be shocks at $V_{ff} \sim 3000$ km/sec which heat some gas in BL to $\sim10^8$ K

In magnetic WDs, accreted fluid channeled to small (~10 km) polar cap with ff velocity, and so can have stronger $T \sim 10^8$ K emission (~10 keV)

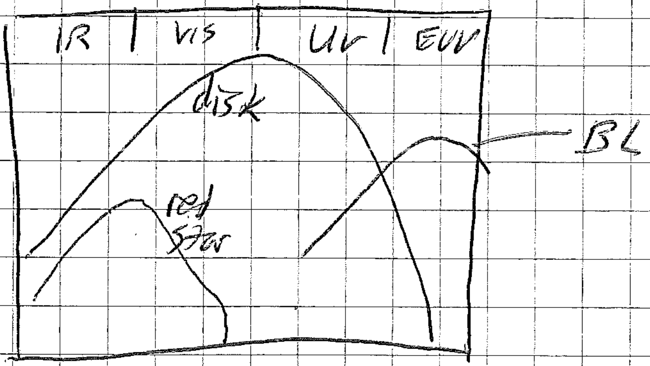
May have cyclotron polarized radiation near cyclotron frequency

$$\nu_B = \frac{eB}{m_e c} = 2.8\times10^{13} \left(\frac{B}{10^7 \text{G}}\right) \text{Hz}$$

ie, near IR. Some visible light also polarized.

Some of heat from shocked gas may be absorbed in surface layers of WD and then re-radiated at the EUV-soft-x-ray bands. ~~characteristic of WD's surface temperature~~.

Typical spectrum:



Classical Novae:

(10^{-5}-10^{-4} $M_\odot$)

When enough H accumulates on WD surface ~~at~~ the base of the H layer reaches ρ, T high enough to ignite thermonuclear detonation powered by $p \to He$. When, L reaches $L_{edd} \sim 10^{38}$ erg/s, the H layer gets blown off at $v \sim 1000$s km/sec, and this limits outburst duration to $\Delta t \sim$ weeks-months.

At $\dot{M} \sim 10^{-9}\ M_\odot/yr$, recurrence times are $\sim 10^4$-10^5 yrs

Dwarf novae:

Outbursts of 2-20 day duration
recurrence times ~ days-decades
luminosities $\sim 10^{34}$ erg/sec

Are not due to thermonuclear explosion because:
① would otherwise runaway to L_{edd}
② Nuclear burning times are too long
③ Recurrence times too long

Dwarf novae are due to variations in accretion rate, due to

ⓐ varying the mass transfer rate from the companion

ⓑ varying α,

or both.

ⓐ Suppose we suddenly dump a large mass (ΔM) in short time (e.g., 1 hr) from L_1. Want $\Delta M \sim M_d$ to make $\mathcal{O}(1)$ change in disk.

Mass initially settles (in dynamical time t_ϕ) to ring at radius $R_h \sim$ outer disk radius

Ring then diffuses into a disk, on a viscous timescale, $t_{visc} \sim R^2/\nu \sim R/V_R$, or

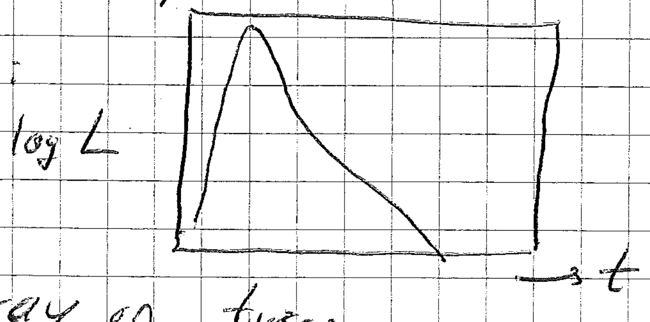
$$t_{visc} \sim \frac{R^2}{\nu} \sim \frac{R}{H}\frac{R}{V_\phi}\frac{V_\phi}{c_s}\frac{1}{\alpha} \sim \frac{1}{\alpha}\mathcal{M}^2 t_\phi,$$

where $t_\phi \sim R/V_\phi$ $\quad \mathcal{M} = \frac{V_\phi}{c_s}$ $\quad \nu = \alpha c_s H$ $\quad \frac{H}{R} \sim \frac{1}{\mathcal{M}}$

Numerically, $t_{visc} \sim 3\times10^5\,\mathrm{sec}\,(\alpha)^{-4/5}\left(\frac{\dot{M}}{10^{16}\,\mathrm{g/s}}\right)^{-3/10}\left(\frac{R}{10^{10}\,\mathrm{cm}}\right)^{5/4}$

$\sim$ days–weeks

Get light curve:



fast rise; decay on t_{visc}.

ⓑ But suppose we start with a disk with very low α and then suddenly raise it by large amount.

During low-α phase, matter builds up in ring near R_h. If then turn on α, it then spreads on viscous timescale, as in ⓐ.

①, then ⓐ may occur if there are convective and/or ~~of~~ magnetic instabilities in the envelope of the companion possibly due also to H/He ionization zones.
Timescale ≲ ~~day~~ hour for dynamical change in * is OK, but modeling is difficult.

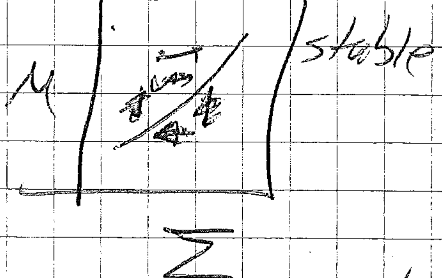
ⓑ May occur from viscous instability in disk

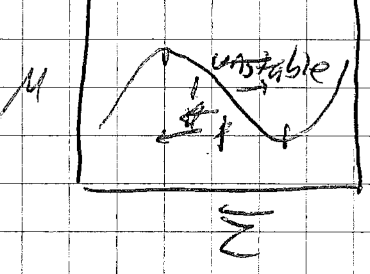
The steady-state thin-disk solution specifies

$$\cancel{\mu \equiv}\ \nu \Sigma = \frac{\dot{M}}{3\pi}\left\{1-\left(\frac{R_*}{R}\right)^{1/2}\right\},$$

but does not distinguish ν from Σ. Define $\mu \equiv \Sigma$, and let $\mu_{ss}(R)$ be the standard result, and let this be $\mu_{ss} = \mu_0(\Sigma_{ss})$; i.e., at each pt. in disk there is reln between local value of ν and Σ: $\nu(\Sigma)$

Suppose $\mu_0(\cancel{\Sigma})$ $\frac{d\mu_0}{d\Sigma} > 0$. Then ~~Σ~~ if the microphysics ~~dictates~~ that is disturbed so that μ is high, then ~~Σ~~ matter in that ring ~~accumulates~~ depletes accumulate ~~increasing~~ decreasing increasing Σ and putting us back on the curve; and vice versa for low μ

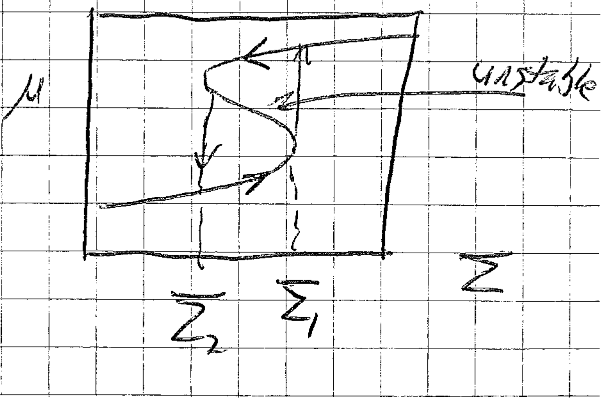




But if $d\mu/d\Sigma < 0$, then if μ is high, accumulation of matter takes us away from curve.

Suppose, though, that $\mu(\Sigma)$ is double-valued,

and suppose that the thin-disk soln tells us that $\mu_{ss}(R)$ falls on the unstable branch



That soln cannot exist in steady state.
But suppose we start on the lower branch with $\Sigma_2 < \Sigma < \Sigma_1$ ~~at~~ that point in the disk.

Since $\mu(R_0) < \mu_{ss}(R_0)$ there soln evolves to higher μ and higher Σ on lower branch until Σ reaches Σ_1. It then jumps to the upper branch and evolves down to Σ_2 and jumps back down to the lower branch......

$\Rightarrow$ Limit cycle alternating between high μ and low μ.

Since ν is controlled by microphysics, it ~~should~~ can change on a thermal timescale,

$$t_{th} = \frac{\text{heat content/area}}{\text{dissipation rate/area}} \sim \frac{\Sigma c_s^2}{D(R)}$$

and $D(R) \sim \nu \Sigma \frac{GM}{R^3}$, so

$$t_{th} \sim \frac{R^3 c_s^2}{\nu G M} \sim \frac{c_s^2}{v_\phi^2}\frac{R^2}{\nu} \sim \mathcal{M}^{-2}\, \overset{t_{visc}}{\cancel{th}}_{visc} \ll \cancel{t}\; t_{visc}$$

$$\sim \frac{1}{\alpha} t_\phi$$

Regions of high and low viscosity ~~are sepa~~ in the disk ~~are~~ separated by an annular width ΔR

$$\frac{(\Delta R)^2}{\nu} \simeq t_{th} \leftarrow$$ viscous ~~diffusion~~ diffusion over distance ΔR on timescale t_{th}

$$\Rightarrow (\Delta R) \simeq (\nu t_{th})^{1/2} \sim \left(\nu \frac{t_\phi}{\alpha}\right)^{1/2} \sim (\alpha c_s H t_\phi/\alpha)^{1/2} \sim H$$

The front between high and low velocity moves at velocity $V_f \simeq \Delta R / t_{th} \sim H/t_{th} \sim \alpha c_s$, so the disk can switch from high to low state in time

$$\Delta t \sim \frac{R}{V_f} \sim \frac{R}{\alpha c_s} \sim \left(\frac{R}{\alpha H}\right)\frac{1}{\Omega} \sim \frac{\mathcal{M}}{\alpha} t_\phi \ll t_{visc}$$

$\Rightarrow$ transition can occur much faster than viscous timescale.

The double-valued $\mu(\Sigma)$ is believed to be associated with H ionization when the disk has $T \simeq 10^4$ K, and particularly, with the large change in opacity.

Thus at given Σ can have

low opacity

① low T, low ν ($\nu \propto c_s H$, $H \propto c_s$, $c_s^2 \propto T$) low $D(R)$ (since $D \propto \nu\Sigma$), and high cooling rate

② high T, high ν, high $D(R)$, high opacity

$\Rightarrow$ This instability only arises in disks with $T \sim 10^4$ K. !!

X-ray binaries:

peak @ 0.1–100 keV

binaries with NS/BH and companion

$\frac{GM}{R}$ for NS such that ~10% of rest-mass energy of accreted material released

up to ~40% for BH

usually have $L \sim L_{Edd} \sim 10^{38}$ erg/s

and $\dot{M} \sim 10^{-8}\, M_\odot$/yr

From $L \simeq 4\pi R^2 \sigma T^4$ $R \sim 10^6$ cm $\Longrightarrow T \simeq 2\times10^7$ K

$\nu = 1.2\times10^{18}$ Hz $\lambda \simeq 25$ Å 5 keV

Max temperature will occur when free-falling matter hits NS surface at $V_{ff} \sim 0.1$

From $\frac{1}{2} m_p V_{ff}^2 = \frac{3}{2} k T_{ff} \Rightarrow T_{ff} = 3.6\times10^{12} \left(\frac{V_{ff}}{c}\right)^2$ K

Usually $T \sim T_{bb}$ rather than T_{ff}

$L \lesssim L_{Edd} \sim 10^{38}$ erg/sec usually

Many NSs highly magnetized (e.g, 10^{12} G), and these can disrupt the usual accretion flow at the magnetic or Alfvén radius R_m at which B-field pressure equals ram pressure of free fall:

$$\rho v^2 = \frac{B^2}{8\pi}.$$

E.g, assuming spherical symmetry, $\dot M = 4\pi r^2 \rho v_{ff}$, $B = \frac{\mu}{r^3}$, $v_{ff} = (2GM/r)^{1/2}$

$$\Rightarrow R_m = 2^{-2/7} \mu^{4/7} \dot M^{-2/7} (2GM)^{-1/7}.$$

Using $L_x = GM\dot M/R \Rightarrow R_m \simeq 1.4\times10^8 (L/L_{edd})^{-2/7} (\mu/10^{30}\,\mathrm{G c}$
$> 10^6\,\mathrm{cm} = R_{NS}$

At $r \lesssim R_m$, fluid flow is channeled along field lines to polar caps. E.g. Electron cyclotron frequency is

$$h\nu_B = 11.6 \left(\frac{B}{10^{12}\,G}\right) \mathrm{keV} \Rightarrow \text{B-field important in radiative processes}$$

$\Rightarrow$ may be observed spectral features; e.g, Her X-1

Misalignment of B field with poles $\Rightarrow$ X-ray pulsations

X-ray Pulse periods are, e.g, in SMC X-1 and A0538-66 at $P = 0.714$ s and $P = 0.069$ sec $\ll$ 1 sec
$\Rightarrow$ sources are NSs, not WDs.

There is also spinup observed due to accretion of matter. ~~Accret~~ Matter accreted from disk has specific angular momentum at R_m of

$h = (GMR_m)^{1/2}$. Rotation frequency Ω changes as

$$\dot L = \dot M h = I\dot\Omega \Rightarrow 2\pi I \frac{\dot P}{P^2} = -\dot M h$$

or $\dot P = -\frac{\dot M h P^2}{2\pi I}$, and using R_m from above,

spin-up time $t_s \cong \frac{P}{\dot P} = 2^{17/14} \pi I P^{-1} \dot M^{-6/7} \mu^{-2/7} (GM)^{-3/7}$

Measurements of P-$\dot{P}$ $\Rightarrow I \sim 10^{45}$ g-cm^2 $\sim I_{ns} \ll I_{wd}$
identifying NSs as compact object, even in x-ray
to systems with $P \gtrsim$ min

Some sources (generally non-pulsating and/or weak B) exhibit
Type-I x-ray bursts, in which $L_x \uparrow$ by ~10 over ~~10s~~
~10s seconds with recurrence times ~10 hours.
These are due to nuclear ignition of matter accreted
onto surface of star — more later.
Spectrum during decay of XRB ~ BB at fixed
$T \Rightarrow$ infer $R_{ns} \sim 10$ km. instabilities in
Type II XRBs: ~ dwarf novae; accretion disk
Low-mass x-ray binaries (LMXBs) are those in
which matter accreted onto NS or BH from Roche-lobe
overflow from low-mass companion. (e.g., 1822-37)

E.g., "Galactic bulge sources"

Also seen in ~~GCs~~ globular clusters. In GCs,
stars may ~~[illegible]~~ scatter gravitationally from each
other. If so, then more massive stars settle to
the center of the GC. The observed distribution of
LMXBs in GCs is similar to the stellar distn.
suggesting that ~~these~~ the LMXBs have masses ~$M_\odot$

The high rate of stellar encounters in a GC may
enhance the rate of formation of binaries leading
to larger ~~[illegible]~~ abundance of LMXBs therein. However
may suggest that this might be counteracted by
escape of NSs upon birth given that typical kick
velocities are small compared with the GC escape velocity

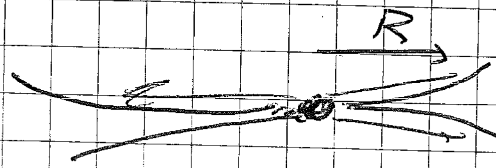
High-mass x-ray binaries (HMXBs): have massive
stellar (e.g., OB star) companion. ~~#~~ NS/BH accretes
matter from stellar wind. E.g., Cyg X-1 Cen X-3 ~~Her X~~1.

Are highly variable due to changes in wind, or absorption
may have regular variability if $\exists$ eccentric orbit

Many are BH candidates

Centaurus X-3: $P \sim 4.8$ s & 2.087-day orbit
O6 companion; very circular orbit, just outside stellar photosphere. Has extended off states

Irradiated Disks:



Since thin disk may be flared, may be irradiated (i.e. heated) by radiation from central source, even if that source is ~point source.

If pt source radiates at temperature T_e defined by $L_{pt} = 4\pi R_*^2 \sigma T_e^4$, the temperature of disk resulting from irradiation at radius R is

$$\left(\frac{T_{pt}}{T_e}\right)^4 \sim \frac{H}{R}\left(\frac{R_*}{R}\right)^2 C \qquad C \lesssim 10^{-2}-10^{-1}$$

geometrical factor

So $T_{pt} \propto R^{-1/2}$ as opposed to $T_{eff} \propto R^{-3/4}$ for temperature of disk due to viscous heating.

From $T_{eff} \simeq \left(\frac{3GM\dot{M}}{8\pi R^3 \sigma}\right)^{1/4}$ and $L_{pt} = GM\dot{M}/R_*$,

$$\Rightarrow \left(\frac{T_{pt}}{T_{eff}}\right)^4 = \frac{2}{3}\frac{H}{R_*}\frac{R}{R_*}C.$$

e.g., in LMXBs $R_* \sim 10^6$ cm and $R_{out} \gtrsim 10^{10}$ cm, so

$T_{pt} \gtrsim T_{eff}$ even if $CH/R \lesssim 10^{-3}$.

May also have heating from extended source (e.g. WD in CV), inner part of accretion disk (e.g., with BH), or accretion corona..... many possible complications.

One consequence:

If irradiation heats disk to $T_{pt} > T_H \sim 10^4$ K ~~at~~ all the way out to R_{out}, thermal instability that gives rise to dwarf novae (Type II XRBs) may be suppressed.

Observationally, only systems with $T_{pt}(R_{out}) < T_H$ show outbursts; soft x-ray transients (SXTs).

Condition $T_{pt}(R_{out}) < T_H$ (or analogous constraint for irradiation by inner edge in BHs) is even stronger (by ~50) constraint to $\dot{M}$ than dwarf-nova condition $T_{eff}(R_{out}) < T_H$. Find that SXTs have max accretion rates,

$$\dot{M}^{max}_{SXT} \simeq \left\{ \begin{matrix} 5\times10^{-11} \\ 5\times10^{-10} \end{matrix} \right\} M_1^{2/3} \left(P/3hr\right)^{4/3} M_\odot/yr.$$

Since BH constraint is weaker (and since $\dot{M}$ generally lower with the larger primary mass), is reasonable to expect that many SXTs have BHs, and this seems to be true. Is taken as evidence for existence of horizon in BHs.