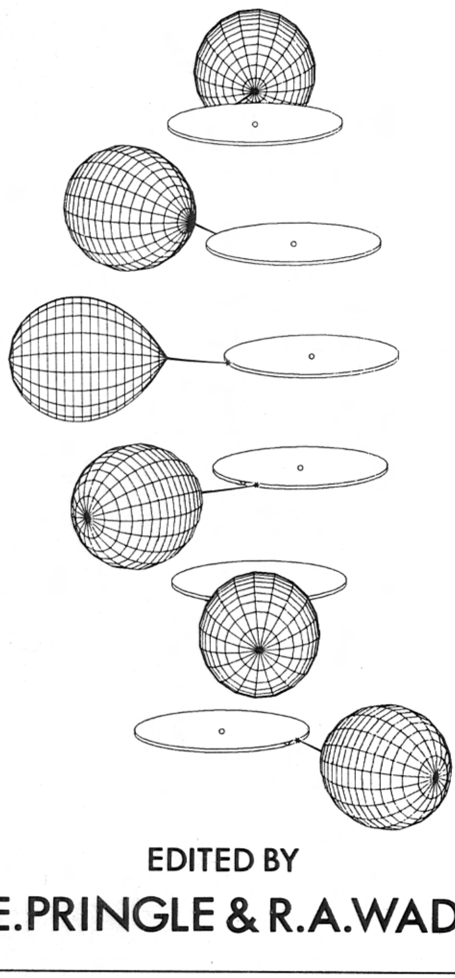
Interacting binary stars



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2.2

STELLAR EVOLUTION AND BINARIES

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2.2.1 Introduction

The evolution of a star in a close binary system differs from that of an isolated star because of the proximity of its companion star, which limits the extent to which it can grow. Many manifestations of the distortion of one star's structure or appearance by the presence of a close companion have been known for the better part of a century now (for example, the reflection effect, and apsidal motion) but, in large measure, we owe the realization that the presence of a close companion may fundamentally alter the course of evolution of a star to the work of Otto Struve. In 1941, he deduced for the first time the existence of gaseous streams between the components as an explanation for the peculiar behaviour of the spectrum of β Lyrae through eclipse, and over the course of the next decade demonstrated similar phenomena in a wide variety of other eclipsing binaries. The publication of Struve's interpretation of β Lyrae was accompanied by a theoretical paper by Gerard Kuiper, in many respects far ahead of its time, discussing for the first time the dynamical behavior of gas streams, pulled from one star by the tidal action of a companion, and the physical interaction between two stars in actual contact. The full significance of Struve's and Kuiper's work did not become apparent until the recognition, a decade later, of the famous Algol paradox (that the more evolved star is less massive in these binaries), and its resolution in terms of an earlier phase of large-scale mass transfer between the two components of these eclipsing binaries, as first proposed in 1955 by John Crawford. Close binary evolution assumed an existence of its own.

In the past 30 years, our theoretical and observational understanding of close binary evolution has itself been developed in considerable detail. To be sure, many phenomena, even quite important ones, still have no secure explanation, but it is possible to outline in a broad perspective the course of close binary evolution. To do so, however, we must first understand the driving forces in the evolution of single stars.

2.2.2 The evolution of isolated stars

In anthropomorphic terms, the evolution of a single star may be considered a struggle in which the star attempts to sustain itself against its own self-gravity by calling upon successive nuclear-burning cycles. The most important of these cycles consume hydrogen, helium, and carbon as fuels. (Of course, reactions involving other elements also occur, but while they are crucial to understanding nucleosynthesis in stars, they are generally of minor importance in determining the structure of a given star or the direction in which it evolves.) Except for the hydrogen-burning reactions which power main sequence stars, each major cycle consumes as its fuel the products of the preceding cycle, and is ignited at or near the center of the star, where the high density and temperature most favor nuclear reactions.

As each fuel is exhausted in the central region of a star, the nuclear reactions which had supported that core by replenishing its energy losses to the outer layers of the star die out. The core is then gradually compressed by the weight of the overlying layers of the star, further increasing its density, and the site of the latest cycle of nuclear burning is transposed from the center of the star to a shell surrounding the core. Compressional work is done on the core, but it continually loses energy to the envelope of the star by radiative heat transport and the specific entropy of matter in the core therefore slowly decreases. Its central temperature may rise or fall, depending on how near the core is to electron degeneracy.

Ultimately one of two situations develops: either (a) the core becomes hot and dense enough to ignite the next nuclear reaction cycle, or else (b) the specific entropy of the core falls so low that the free electrons within it become degenerate. In the first case, the transition to a new central energy source occurs quasistatically, and the adjustment required of the star at ignition is typically a moderate one. In the second case, however, the development of a compact degenerate core leads to vigorous nuclear burning in the fuel-rich shell (or shells) surrounding it. The outer envelope of the star responds to this increased energy output by expanding and developing a deep surface convection zone, which reaches inward nearly to the outermost nuclear shell-burning source. This expansion drives the star to the giant branch in the Hertzsprung–Russell diagram (Figure 2.1.2).

As the nuclear-burning shells advance outward, their reaction products are added to the core. The shell sources must burn ever more vigorously to support the envelope above the growing core and the star thus increases in luminosity, expanding and ascending the giant branch as it does so. Meanwhile, the degenerate core is heated by compression and by electron conduction inward from the burning shells at its boundary, and cooled by neutrino losses. It gradually

becomes hotter and denser, and at the same time more degenerate, as it increases in mass. During this degenerate core growth phase, the total stellar luminosity and radius is virtually a function of the core mass alone, without regard to total mass.

Ultimately, either (a) the envelope of the star is exhausted and further core growth is impossible, or (b) ignition conditions are reached for the next round of nuclear reactions, those which consume as fuel the elements constituting the present cores. In the first case, the stellar surface shrinks very rapidly at constant luminosity, becoming hot enough to illuminate briefly the circumstellar material lost in a stellar wind as a planetary nebula; after reaching white dwarf dimensions, the star subsequently cools at constant radius. In the second case, the fact that the pressure of a degenerate gas is quite insensitive to temperature, whereas the temperature dependence of the nuclear reactions being ignited is extremely strong, allows the burning rate in the newly ignited core to accelerate by many orders of magnitude before the gas is heated enough to lift its degeneracy and quench the runaway by expansion.

Degenerate helium ignition is evidently not so violent that it disrupts the star, for we see numerous examples (horizontal branch stars and RR Lyrae variables in very old stellar populations) which have survived it. The core expansion accompanying helium ignition pushes the hydrogen-burning shell surrounding it outward in the gravitational potential well of the star, and as a result of this weakening gravitational confinement the burning rate in the shell decreases. Indeed, the luminosity of the whole star undergoes a dramatic decline, and the stellar radius contracts as well. This phenomenon is apparent in the large drop, from $\sim 10^3\ L_\odot$ to $\sim 10^2\ L_\odot$ in the luminosities of the 1 and 2 $M_\odot$ stars whose evolutionary tracks are illustrated above in Figure 2.1.2. The horizontal branch stars and RR Lyrae variables are similar core-helium-burning stars, of somewhat lower total mass, which make blueward excursions from the giant branch during this phase. The core-helium-burning stars do not resume their ascent of the giant branch in earnest until they have exhausted helium in their cores and are in a double-shell-burning phase.

Theoretical models indicate that degenerate carbon ignition is an altogether more violent event. By the time electron degeneracy has given way to a perfect gas law in the ignited core, allowing the core to begin expanding, carbon burning has become so vigorous that the entire binding energy of the star is realeased before it can readjust hydrostatically – the star explodes as a supernova.

It should be clear that two criteria dictate the qualitative evolution of a single star during its active lifetime: (1) What nuclear fuels is the star capable of igniting? (2) Does that ignition occur under degenerate conditions? Fortunately, as noted at the beginning of this chapter, the answers are strong functions only of the mass of the star, in effect dividing the spectrum of stellar masses into a handful of intervals within which stars follow very similar evolutionary paths. The precise divisions between different evolutionary paths are clouded somewhat by the effects of mass loss, which can be severe even for single stars. Threshold initial masses for degenerate ignitions are particularly affected, since those ignitions occur as a rule late in the stellar lifetime, when stellar wind mass-loss rates tend to be their highest. We give estimates of the important threshold masses below; but some word is in order first, regarding the remnants left behind by single stars.

The most massive star which can be completely supported by electron degeneracy is 1.4 $M_\odot$ – the Chandrasekhar mass limit for white dwarfs. If the mass of the degenerate core of a star should grow beyond this limit, the electrons become relativistic and the star is dynamically unstable to collapse to nuclear densities. It is in fact the approach to incipient collapse which is responsible in more massive stars for the degenerate ignition of carbon and heavier elements leading up to iron, the most tightly bound of all nuclei, all of which require ignition densities in excess of 10^9 g cm^{-3} under degenerate conditions. (Note, however, that none of the reactions beyond carbon burning releases enough energy to disrupt the core.) As a result, the degenerate cores of massive stars tend to converge to this limiting mass, a phenomenon which is illustrated strikingly by the fact that, in all cases where mass estimates have been possible (in the original binary radio pulsar and in the binary pulsing X-ray sources), the masses of the neutron stars are consistent with this threshold, 1.4 $M_\odot$.

The theoretical upper mass limit for neutron stars is much less certain, owing to our still inadequate knowledge of the properties of matter at nuclear densities. The best current estimates put it in the neighborhood of 2.0 $M_\odot$. However, it is generally supposed that the collapse of a stellar core to neutron star dimensions ejects the entire outer envelope of the star in a supernova explosion, a supposition which is underscored by observed neutron star masses themselves, as noted above. For a single star to produce a remnant exceeding the upper mass limit for neutron stars – a black hole – it must therefore develop a core exceeding 2 $M_\odot$ which is completely exhausted of any nuclear fuel. Because of the core convergence phenomenon discussed above, core masses this large are only feasible if the core remains non-degenerate throughout all of the nuclear-burning cycles leading up to iron, a scenario which requires a very massive star indeed.

In Table 2.2.1, we list the threshold masses which thus determine the evolutionary path followed by single stars of normal (solar) composition. They are listed in increasing order of their corresponding initial, main sequence masses. The first threshold listed in fact defines the lower mass limit of the main sequence. As noted earlier in this chapter, stars below initial mass ~0.8 $M_\odot$

cannot evolve beyond the main sequence within the age of the universe, and so the second threshold is evidently exceeded by all stars capable of evolving to their remnant state within this time.

In exploring close binary evolution, our interest will focus on the growth of stars in radius, since this determines the point at which tidal mass transfer may occur. In Figure 2.2.1, the radii of evolving single stars have been plotted as a function of stellar mass. As discussed above, the episodes of most dramatic growth clearly correspond to transitions from core to shell burning and to phases of degenerate core growth, and it is therefore during these phases that mass transfer is most likely to be initiated in a binary system.

2.2.3 The mass-losing star

When a star fills its tidal (or Roche) lobes, the fundamental constraints on its structure are altered. No longer does the star evolve at constant mass

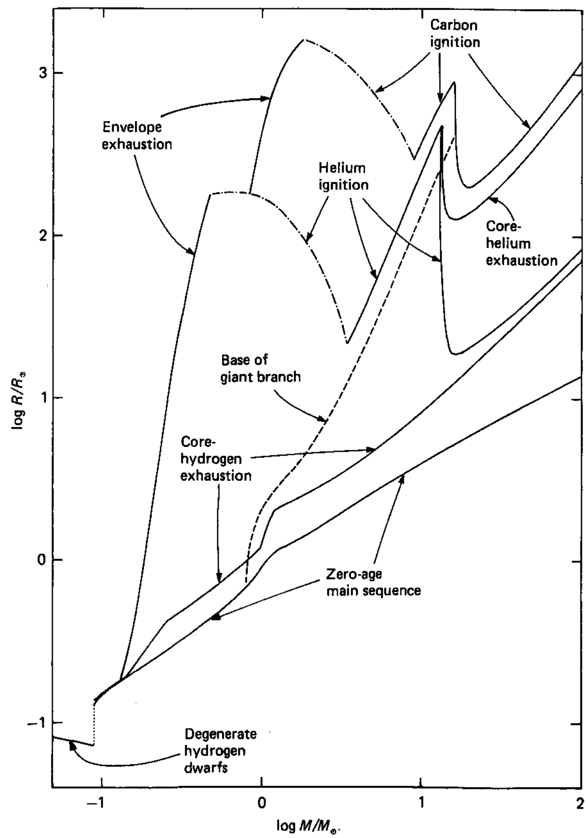
Table 2.2.1. *Mass thresholds in single stars*

Threshold	Approximate main sequence mass	Core mass	Remnant
			hydrogen white dwarf
minimum, non-degenerate hydrogen ignition	0.085 $M_\odot$	0.085 $M_\odot$	
			helium white dwarf
minimum, degenerate helium ignition	0.7 $M_\odot$	0.47 $M_\odot$	
minimum, non-degenerate helium ignition	2.25 $M_\odot$	0.31 $M_\odot$	carbon-oxygen white dwarf
minimum, degenerate carbon ignition	$>$4.0 $M_\odot$	1.4 $M_\odot$	
maximum, white dwarf	$>$4.0 $M_\odot$	1.4 $M_\odot$	none?[a]
minimum, non-degenerate carbon ignition	10.0 $M_\odot$	1.06 $M_\odot$	
			neutron star[a]
maximum, neutron star	$>$30.0 $M_\odot$	2.0 $M_\odot$	
			black hole

[a] Stars as low as 8 $M_\odot$ ignite carbon under sufficiently mildly degenerate conditions to avoid disruption.

(excepting stellar wind losses from its surface), but it is now obliged to maintain its radius within a close tolerance of that of the Roche lobe, losing mass if it must do so – or else pay the price in enormous mass-loss rates if it cannot so restrain its surface. The direction of evolution of the star, and of the binary

Fig. 2.2.1. Radii of stars of solar composition at various stages of their evolution. Degenerate helium and carbon ignition are differentiated by dash–dotted lines (after R. F. Webbink, 1979, in *IAU Colloquium 53, White Dwarfs and Variable Degenerate Stars*, eds. H. M. Van Horn & V. Weidemann, Rochester: Univ. Rochester Press, p. 426).



system, thus depend on whether the star can 'track' the evolution of its Roche lobe with its own stellar radius and, if so, whether it can accomplish this feat without sacrificing its own thermal equilibrium.

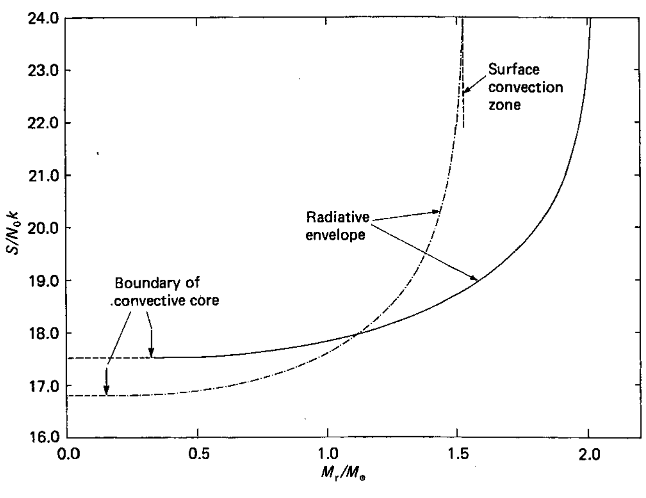
A crucial question is this: How do stars readjust internally to the loss of mass? In answering this question, we should first recognize that stars are very centrally condensed objects. Indeed, it is this concentration of mass near the centers of stars which justifies the use of Roche potentials in the first place, as noted in Chapter 1. It follows, therefore, that since it is the outer, tenuous layers of a star which are liable to be stripped away tidally by a companion, even mass-loss rates, which we label 'dynamical' below, do not disturb hydrostatic equilibrium in the stellar interior to a great extent. The interior responds on its own dynamical timescale, of order $(G\langle\rho\rangle)^{-1/2}$, which is much shorter than the sound travel times characterizing mass loss in the unstable envelope, which are of order $(G\langle\rho\rangle\Delta R/R)^{-1/2}$. (Here $\langle\rho\rangle$ is the average density interior to a point in the star, and $\Delta R/R$ is the fraction by which its radius exceeds that of its Roche lobe.) Thus, we do not expect hydrostatic equilibrium to break down in the interior of a star losing mass in a binary system, but the star's thermal equilibrium – the balance between its nuclear energy sources and its surface radiation losses – may well do so.

In examining the possibility of the breakdown of thermal equilibrium within a star, a useful quantity to consider is the entropy per unit mass of stellar matter as it varies through the stellar interior. The reason for doing so is that, in the limit of rapid changes in the mass of the star, the entropy profile in the stellar interior remains unchanged, that is, the star responds adiabatically. Further, by comparing the entropy profile in a star at the onset of mass loss with that at a later point, we see immediately what regions of the star must have absorbed energy in the process (specific entropy increases) and what regions must have released energy (specific entropy decreases).

As an illustration, Figure 2.2.2 shows the entropy profile in an unevolved 2.0213 $M_\odot$ star of solar composition, radius 1.495 $R_\odot$. This star has a deep radiative envelope and a convective core. The criterion for convection to occur is that an element displaced upward (downward), maintaining pressure equilibrium with its surroundings in the star, must be less (more) dense than the surrounding medium; that is, it must have a higher (lower) temperature and hence higher (lower) specific entropy than the ambient medium. Clearly, then, regions which are stable against convection must have specific entropy increasing outwards (positive entropy gradient), whereas those with specific entropy decreasing outwards (negative entropy gradient) are unstable to convection. In fact, except very near the stellar surface, convection is normally so efficient that only a miniscule negative entropy gradient is sufficient to carry the entire stellar energy flux in the presence of convection, and thus the convective core of the star in Figure 2.2.2 has virtually constant specific entropy. Still, the division into convective core and radiative envelope is clear from its entropy profile. Note that in the outer radiative envelope of the star, the specific entropy rises very rapidly toward the surface (off scale in Figure 2.2.2): the pressure and temperature scale-heights of the star are smallest near the surface, and hence the specific entropy varies particularly rapidly there. This star actually has two very thin, inefficient convection zones near its surface which produce a sharp drop in entropy (off scale in Figure 2.2.2).

Also shown in Figure 2.2.2 is the entropy profile of an unevolved 1.5249 $M_\odot$ star of the same composition, radius 1.276 $R_\odot$. Imagine it to be the remnant, following mass loss in a binary system, of the 2.0213 $M_\odot$ star. It is clear, then, that the matter in the outer 0.40 $M_\odot$ of the remnant star must have absorbed a considerable amount of energy in reaching its new state, because its specific entropy is dramatically higher than that of the parent star in this mass range. Similarly, within the central 1.12 $M_\odot$ of the star the specific entropy of the remnant is lower than that of the parent, and so this region releases energy in

Fig. 2.2.2. Specific entropy profiles in the interiors of a 2.0213 $M_\odot$ and a 1.5249 $M_\odot$ zero-age main sequence star of solar composition (solid and dash-dotted lines, respectively). Convectively unstable regions are indicated by dashed lines.



the transition. The crossing point at $M_r = 1.12\ M_\odot$ occurs at very nearly the same radius in both stars (0.375 $R_\odot$ in the 2.0212 $M_\odot$ star, and 0.385 $R_\odot$ in the 1.5249 $M_\odot$ star). Thus, most of the volume of the star (98.4 per cent of the parent star) is contained in the high-entropy envelope.

Suppose now that the process of mass loss from 2.0213 $M_\odot$ to 1.5249 $M_\odot$ occurred on a rapid timescale, faster than the interior of the star could relax to thermal equilibrium. Then, throughout the bulk of the stellar envelope, matter finds itself with much lower entropy than it should have in the thermal equilibrium state appropriate to its new mass (in this case, a main sequence star): it is therefore much denser and cooler than normal, and the star will be smaller in radius and much lower in luminosity than a corresponding main sequence star, absorbing most of the energy outflow from its interior as it strives to regain thermal equilibrium. It is this ability of the star to shrivel beneath its normal dimensions which enables such a star to track its Roche lobe when it contracts rapidly at the beginning of mass transfer, and it must be stressed that this ability hinges on the star having a radiative envelope, that is, one in which specific entropy rises rapidly toward the surface.

In the central regions of the star where nuclear burning supplies the normal stellar luminosity, matter finds itself with a higher specific entropy than it should have in its new state. Because the weight of the surface layers has been lifted, the higher entropy of the core actually allows it to expand and cool further than it would in thermal equilibrium, thus in effect shutting down core nuclear burning. Only as the core returns to thermal equilibrium, dropping in specific entropy, can its density and temperature grow large enough to permit nuclear burning to resume at a rate appropriate to its new mass.

Clearly, the response of a star to rapid mass loss is intimately interwoven with its internal thermal structure. The behavior of the stellar radius depends sensitively on the radiative or convective stability of its outer envelope. Had we chosen an example above with a deep surface convection zone, the stellar radius would have reacted far differently to decreasing mass. In fact, deep convective envelopes tend to produce net *expansion* rather than contraction in response to mass loss. It is easily shown, for example, that the radius of a star in hydrostatic equilibrium which is adiabatically convective throughout (in which, therefore, pressure varies with density as $P \propto \rho^{5/3}$ in the case of a perfect gas), varies with its mass as $R \propto M^{-1/3}$. Non-relativistically degenerate white dwarfs (zero-entropy stars) obey similar P-ρ and R-M relationships. These stars may thus find it impossible to contain themselves within their Roche lobes if the lobes contract in response to mass transfer.

Mass transfer in a binary system can be triggered either by the evolutionary expansion of a star until it fills its tidal lobe, or by orbital contraction resulting from orbital angular momentum losses. How the system will evolve from that point onward, and, in particular, what rates of mass transfer may be expected, depends upon the relationship between three parameters:

$\zeta_L \equiv \dfrac{d \ln R_L}{d \ln M}$, the logarithmic derivative of the tidal (Roche) lobe radius with respect to the mass of the lobe-filling star;

$\zeta_e \equiv \left(\dfrac{\partial \ln R_{eq}}{\partial \ln M}\right)_X$, the logarithmic derivative of the radius of a star in thermal equilibrium, with a fixed composition profile, with respect to its mass; and

$\zeta_s \equiv \left(\dfrac{\partial \ln R}{\partial \ln M}\right)_{X,s}$, the logarithmic derivative of the radius of a star with respect to its mass, for a fixed initial profile in composition and specific entropy.

We will refer to ζ_L, ζ_e, and ζ_s as the tidal, equilibrium, and adiabatic radius–mass exponents, respectively.

Three cases may be distinguished on the basis of the ordering of these derivatives: (1) If $\zeta_s < \zeta_L$, then as the lobe-filling star loses mass ($d \ln M < 0$), it cannot remain within its Roche lobe no matter how rapidly it loses mass. This condition we label *dynamical timescale* mass transfer, because the mass-loss rate from the lobe-filling star is limited only by the sonic expansion rate of its envelope through the gravitational nozzle at the inner Lagrangian point (L_1), as discussed in Chapter 1. (2) If $\zeta_e < \zeta_L < \zeta_s$, the lobe-filling star cannot remain in thermal equilibrium as it loses mass, since its radius would then overfill its Roche lobe by an ever-increasing amount, leading to mass-loss rates too high to permit the star to remain in thermal equilibrium. In that limit, however, the radius would follow the adiabatic relationship characterized by ζ_s, and the star would contract within its Roche lobe. In fact, it is the divergence from thermal equilibrium which permits the star to remain just filling its Roche lobe, and the mass-loss rate is therefore regulated by a *thermal timescale*, rather than by a dynamical one. (3) The final case arises if $\zeta_L < (\zeta_s, \zeta_e)$. Then mass loss proceeds not by virtue of the inability of the star to remain in equilibrium (dynamical or thermal) within its Roche lobe; that is, mass transfer is not self-stimulating. Instead, it occurs only by virtue of the evolutionary expansion of the lobe-filling star, or because orbital angular momentum losses (*other* than those produced in direct consequence of tidal mass transfer itself) draw the Roche lobe inexorably inward upon the star. In this case, mass transfer proceeds on a *nuclear or orbital evolutionary timescale.* We will discuss each of these three cases in turn.

(1) Dynamical timescale mass transfer

As noted in the discussion of the structural readjustment of stars in response to mass loss, the adiabatic radius–mass exponent, ζ_s, of a star depends sensitively on the existence and depth of a surface convection zone. For stars with fully radiative envelopes in thermal equilibrium, ζ_s can become extremely large, and such stars are stable against dynamical timescale mass transfer in a binary system. (It is possible, however, for the dynamical timescale instability to develop in consequence of thermal timescale mass loss depressing the value of ζ_s.) On the other hand, stars with deep surface convection zones, and degenerate stars, have small or even negative values of ζ_s, as noted above, and therefore tend to be unstable to dynamical timescale mass loss unless they are rather less massive than their mass-gaining companions, when it is possible for ζ_L to be even more negative than ζ_s (see the discussion in Chapter 1 of the response of orbital parameters to mass transfer). Conditions for dynamical instability may therefore be realized (i) if the lobe-filling star lies on or very near the giant branch, (ii) if it lies on the lower main sequence, or (iii) if it is degenerate.

The hydrodynamic problem of mass loss through the region surrounding the inner Lagrangian point was discussed in Chapter 1. For the types of stars which we expect to be susceptible to dynamical timescale mass loss (which have envelopes reasonably approximated by a polytropic equation of state: $P \propto \rho^{5/3}$), the mass-loss rate as a function of $\Delta R/R$ (the fractional radius excess beyond the Roche lobe) is roughly

$$\dot{M} \cong -A \frac{M}{P} \left(\frac{\Delta R}{R}\right)^3, \qquad (2.2.1)$$

where M is the mass of the lobe-filling star, P is the orbital period, and A is a dimensionless coefficient depending on the interior density distribution of the star and (weakly) on the mass ratio of the binary. Typically, A is of order 10–20. Clearly, if $\Delta R/R$ is allowed to grow to more than a small fraction, as it inevitably does in dynamical timescale mass loss, $\dot{M}$ can in principle reach enormous values, well in excess of the ability of any accreting companion star to accommodate. Typically, the mass-loss rate from the lobe-filling star accelerates rapidly once it has, through evolutionary expansion or orbital evolution, grown large enough to begin forcing it out of thermal equilibrium. In this early dynamical phase, $\dot{M}$ varies with time approximately as

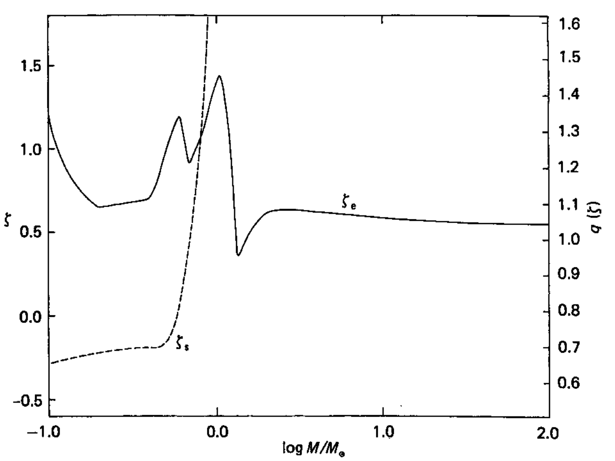
$$\dot{M} \cong -M \left[\frac{8A}{P} \left(\zeta_L - \zeta_s\right)^3\right]^{-1/2} (t_0 - t)^{-3/2}, \qquad (2.2.2)$$

where t_0 is a *future* reference time ($t_0 > t$).

(2) Thermal timescale mass transfer

As might be anticipated from the dicussion of dynamical timescale mass transfer, thermal rather than dynamical timescale mass loss prevails among those non-degenerate stars lying to the left of the giant branch in a Hertzsprung-Russell diagram if they are unstable to rapid mass exchange. The equilibrium radius-mass exponent, ζ_e, for zero-age main sequence stars is of course the very same exponent as that characterizing the zero-age main sequence itself. This exponent is plotted in Figure 2.2.3 for solar-composition zero-age main sequence stars, along with their adiabatic exponents, ζ_s. The right-hand scale indicates the critical mass ratios, $q(\zeta_L)$ = mass of lobe-filling star/mass of accreting star, at which $\zeta_L = \zeta$ in the conservative approximation (conservation of total mass and orbital angular momentum during mass exchange). As a star evolves, ζ_e typically decreases slowly, approaching zero or even becoming negative as the star reaches the base of the giant branch, so that longer-period binaries with a given mass and mass ratio are slightly more susceptible to instability than their shorter-period counterparts. As can be seen from Figure 2.2.3, unless the stars are of nearly equal mass, conditions for thermal timescale

Fig. 2.2.3. The equilibrium (ζ_e) and adiabatic (ζ_s) radius–mass exponents of zero-age main sequence stars of solar composition. The right-hand scale marks the critical mass ratio (lobe-filling/accreting star) at which the Roche lobe radius–mass exponent (ζ_L) equals the corresponding value of ζ at constant total mass and orbital angular momentum.



mass loss will be satisfied in the usual case where the more massive star is the first to fill its Roche lobe, provided it is not pre-empted by dynamical instability.

Because the mass-loss rate is ultimately regulated by the thermal disequilibrium imposed on the envelope of the star by mass loss itself, it typically saturates at a rate

$$\dot{M}_{max} \cong -\frac{M}{\tau_{KH}}, \qquad (2.2.3)$$

where τ_{KH} is the initial thermal (Kelvin-Helmholtz) timescale of the lobe-filling star:

$$\tau_{KH} \cong \lambda \frac{GM^2}{RL}, \qquad (2.2.4)$$

where λ is a coefficient of order unity ($\lambda \cong \frac{1}{3}(L/L_\odot)^{1/8}$ gives a good approximation to $\dot{M}_{max}$ in detailed models). The approach to this peak mass transfer rate is characterized by an exponential rise in $\dot{M}$, $\dot{M} \propto \exp(t/\tau_{\dot{M}})$, where the time constant, $\tau_{\dot{M}}$, can be reasonably well approximated as

$$\tau_{\dot{M}} \cong \frac{\tau_{KH}}{20(\zeta_L - \zeta_e)}. \qquad (2.2.5)$$

(3) Nuclear or orbital evolutionary timescale mass transfer

The final possibility is that mass transfer is driven purely by evolutionary processes (as opposed to the self-stimulated transfer of the previous two cases). The radius of the lobe-filling star remains for all practical purposes constrained identically to that of its Roche lobe, as in thermal timescale mass transfer, but in this case the star will normally remain in thermal equilibrium - although there are exceptions when the star or its orbit is itself evolving very rapidly.

It should be clear from Figure 2.2.3, for example, that conditions for mass transfer on an evolutionary timescale are realized at the onset of the first phase of mass transfer in a binary only if the two stars are nearly equal in mass. They occur frequently, however, after an initial episode of mass transfer when the lobe-filling star has been whittled to a smaller mass than its companion, and its Roche lobe ceases its rapid contraction (ζ_L becomes small or negative). Because mass transfer on an evolutionary timescale is much more protracted than in other cases, it is widely observed among the most common types of binaries, for example in the Algol binaries and among cataclysmic binaries.

Assuming the lobe-filling star remains in thermal equilibrium, we can obtain a simple expression for the mass transfer rate simply by demanding that the star, of radius R, continuously fill exactly its Roche lobe, of radius R_L; that is, that $R = R_L$ and $\dot{R} = \dot{R}_L$. The rate of change of the stellar radius may be written

$$\dot{R} = \left(\frac{\partial R}{\partial t}\right)_{\dot{M}_{\rm tid}=0} + \frac{R}{M}\left(\frac{\partial \ln R_{\rm eq}}{\partial \ln M}\right)\dot{M}_{\rm tid}, \qquad (2.2.6)$$

where the first term on the right-hand side refers to the evolutionary rate of increase due to nuclear evolution in the absence of tidal mass loss. (We distinguish here between tidal and other forms of mass loss because stellar winds, for example, may also play an important role on nuclear timescales.) The time derivative of the Roche lobe radius may likewise be written as

$$\dot{R}_{\rm L} = \left(\frac{\partial R_{\rm L}}{\partial t}\right)_{\dot{M}_{\rm tid}=0} + \frac{R_{\rm L}}{M}\left(\frac{{\rm d} \ln R_{\rm L}}{{\rm d} \ln M}\right)\dot{M}_{\rm tid}, \qquad (2.2.7)$$

where the first term reflects the evolution of the Roche lobe radius in the absence of tidal mass transfer - due, for example, to changing mass ratios and orbital angular momentum losses resulting from stellar winds. Recognizing the definitions of $\zeta_{\rm e}$ and $\zeta_{\rm L}$, we can then write

$$\dot{M}_{\rm tid} = -\frac{M}{(\zeta_{\rm e}-\zeta_{\rm L})}\left[\left(\frac{\partial \ln R}{\partial t}\right)_{\dot{M}_{\rm tid}=0} - \left(\frac{\partial \ln R_{\rm L}}{\partial t}\right)_{\dot{M}_{\rm tid}=0}\right]. \qquad (2.2.8)$$

The evolutionary growth rate of a single star, $(\partial \ln R/\partial t)$ in this expression, is generally an increasing function of time since, apart from core burning phases, as discussed in Section 2.2.2 above, the stellar luminosity and hence rate of nuclear evolution increase with time. This is illustrated in Figure 2.2.4, in which these parameters are plotted as functions of time for a 1 $M_\odot$ star of solar composition.

Several mechanisms for spontaneous mass and angular momentum loss can potentially contribute to the term $(\partial \ln R_{\rm L}/\partial t)$. These include, for example, stellar winds, the general relativistic radiation of gravitational waves, and spin-orbit exchange of angular momentum. However, we will postpone the discussion of these mechanisms until Section 2.2.5 below.

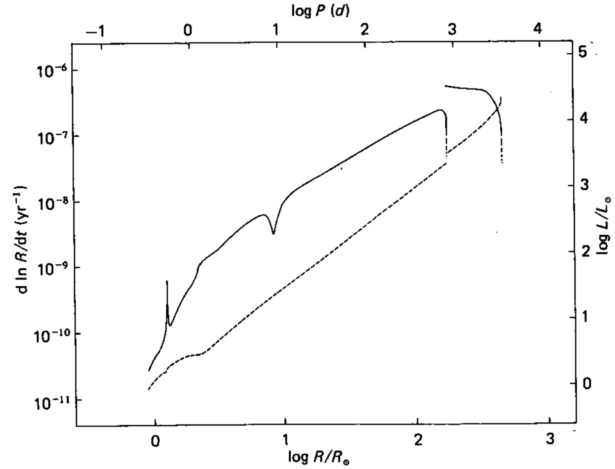
2.2.4 The accreting star

While tidal mass loss can be a traumatic experience for a star if it occurs on a thermal or dynamical timescale, the accretion of this material by its companion star can be devastating. Because the more massive star has the shorter nuclear lifetime and is therefore the first to evolve to fill its Roche lobe, the accreting star is normally then the less massive, less luminous component (at least at the beginning of mass transfer). It is therefore also the star with the longer intrinsic thermal relaxation timescale (see equation (2.2.4)), and its thermal response to its increasing mass departs even further from equilibrium than that of the donor star.

The response of a star to rapid mass accretion is in many respects the inverse of that to rapid mass loss outlined in the previous section. Stars with deep radiative envelopes expand far beyond the equilibrium radii appropriate to their increased mass, as the material being buried beneath the surface has much higher entropy than that deeper in the envelope would have in thermal equilibrium. Similarly, studies of accreting convective stars indicate that they expand less rapidly than an equilibrium sequence of increasing mass, and may even contract.

There are, however, several points at which this simple reflection symmetry with mass-losing stars breaks down. For one, material reaching the surface of the star must dissipate the gravitational potential energy of infall, even before it can be assimilated within the star. This means it is quite likely to have higher specific entropy than matter already at the surface of the accreting star. The rapid expansion of a radiative star will undoubtedly be magnified even further; in a convective star, the erection of an entropy barrier at its surface will tend

Fig. 2.2.4. The evolutionary growth rate (solid line) and luminosity (dashed line) of a 1 $M_\odot$ star as a function of its radius. Corresponding orbital periods at which such a star would fill its Roche lobe are labeled across the top of the diagram. Those evolutionary phases in which the star has contracted from a prior maximum radius have been omitted (after R. F. Webbink, S. Rappaport, & G. J. Savonije, 1983, *Astrophys. J.*, **270**, 678). Copyright American Astronomical Society.



to throttle back its energy outflow. As yet, our theoretical understanding of this aspect of accretion is extremely sketchy, and a great deal more remains to be learned.

A second and more insidious difference lies with the accumulation of angular momentum in the accreting star. The matter raining down on its surface does not fall radially upon it, but carries with it a considerable amount of angular momentum. This not only aggravates the mass-loss instability of the lobe-filling star, by draining angular momentum from the orbit of the binary, but can lead to severe difficulties for the accreting star. The latter problem is especially acute if that star lies far enough within its lobe to permit formation of an accretion disc. In that case, the angular momentum it gains in accreting a mass ΔM is approximately

$$J \cong (GMR)^{1/2} \Delta M, \tag{2.2.9}$$

and its rotational kinetic energy is

$$T \geqslant \frac{J^2}{2I} \cong \frac{G(\Delta M)^2}{2k^2 R}, \tag{2.2.10}$$

where I is the moment of inertia of the star, and k is its dimensionless radius of gyration ($k^2 \cong 0.1$, give or take a factor of two, for main sequence stars). The equality is appropriate in equation (2.2.10) in the unlikely case that dissipative processes can keep the star in uniform rotation. Numerical studies of self-gravitating masses have shown, however, that they become unstable to non-axisymmetric perturbations (bar-like modes) when the ratio of rotational kinetic energy to total gravitational potential energy exceeds a critical value, $T/|W| \gtrsim 0.14$. Since the gravitational potential energy of the accreting star is

$$W = -\alpha \frac{GM^2}{R}, \tag{2.2.11}$$

with α of order unity, the accreting star, if unable to find a means of shedding its accreted angular momentum, will become unstable by the time

$$\Delta M/M \gtrsim (0.28\alpha k^2) \cong 1/6. \tag{2.2.12}$$

How real binaries avoid difficulty on this account is largely an unstudied problem at present, but it is clear observationally that accreting stars must be able to accommodate more mass than this. It seems likely that the accreting star must manage to shed its excess angular momentum in the accretion disc (or even to grow such a disc if need be, as U Cephei appears to do), where a tidal couple with the companion star can restore that angular momentum to the binary orbit.

Another limit to the ability of the accreting star to accommodate the influx of matter arises when the accretion rate becomes so large that the dissipation

rate of the infall energy,

$$L = \frac{GM\dot{M}}{R}, \tag{2.2.13}$$

results in the radiation pressure on infalling matter equalling or exceeding gravitational acceleration. This limiting luminosity is referred to as the Eddington limit,

$$L_{\rm Edd} = \frac{4\pi cGM}{\kappa}, \tag{2.2.14}$$

where κ is the opacity of this matter. In most applications, κ is determined by electron scattering in a fully ionized gas, for which $\kappa \cong 0.2\,(1+X)$ cm^2 g^{-1}, where X is the fraction (by mass) of hydrogen in the gas. The Eddington limit is frequently of importance in high-energy phenomena (accretion onto compact objects), which are discussed later in this book. It effectively limits accretion rates (assuming spherical symmetry) to values

$$\dot{M} \lesssim 2.1 \times 10^{-3} (1+X)^{-1} (R/R_\odot)\ \mathrm{M_\odot\ yr^{-1}}. \tag{2.2.15}$$

If the accreting star is a compact object, rather than a normal (non-degenerate) star, nuclear shell burning in one form or another can be ignited. At sufficiently low accretion rates, the accreted layer is able to cool enough to become degenerate before it can burn. Its recurrent ignition then leads to very virorous, possibly hydrodynamical, events. On accreting, white dwarfs with $\dot{M} \lesssim 10^{-9}\ \mathrm{M_\odot\ yr^{-1}}$ (for a solar mass dwarf), hydrogen ignition in this way is responsible for nova outbursts; at somewhat higher rates, $10^{-9}\ \mathrm{M_\odot\ yr^{-1}} \lesssim \dot{M} \lesssim 2 \times 10^{-7}\ \mathrm{M_\odot\ yr^{-1}}$, recurrent hydrogen ignition still occurs, and the star may swell to giant-like dimensions, but its readjustment is milder and hydrostatic. On neutron stars accreting at rates similar to novae, degenerate helium ignition produces type I X-ray bursts. When accretion rates onto these compact objects become large enough to ignite these shells non-degenerately, they burn stably – and in more extreme cases yet, white dwarfs may revert to red giants if accretion rates exceed the core growth rates for giant stars of the same core mass.

2.2.5 Systematic mass and angular momentum loss

The evolution of a binary through mass exchange depends not only on the structure and response of its two stellar components to mass loss or mass accretion, but also on mechanisms which remove mass and orbital angular momentum from the system. Broadly speaking, we can divide these mechanisms into two classes: those which occur spontaneously, independent of tidal mass transfer; and those which are consequential to the mass transfer process itself.

Spontaneous loss mechanisms

As noted in Section 2.2.3, mechanisms of this sort can in principle instigate mass transfer, even in the absence of evolution in either component, and may also play important roles in determining mass transfer rates during slow evolutionary phases. They include stellar winds, gravitational radiation, and spin–orbit angular momentum exchange, each of which we discuss in turn.

Stellar winds frequently dominate the evolution of extremely luminous stars, which may lose mass much more rapidly than they actually consume it by nuclear burning. This is true particularly of novae during outburst, and of Wolf-Rayet binaries, both of which have dense, optically thick outflowing atmospheres. Winds are also important for binaries of relatively short period ($\lesssim 10$ d) containing cool, convective components, in which exceptionally strong magnetic fields may be generated. Corotation of the stellar wind imposed by the magnetic field is an extremely efficient angular momentum loss mechanism, and is no doubt responsible for the present slow rotation of the Sun, for example. It may prove the dominant ingredient in the evolution of W Ursae Majoris systems and cataclysmic binaries, but unfortunately we have as yet no theory capable of predicting the magnitude of this effect, and a considerable extrapolation is involved in applying observational estimates derived from cool, rotating single stars to their much more rapidly rotating binary counterparts.

General relativistic radiation of gravitational waves is also potentially important for very short-period binaries. Orbital angular momentum, J, is lost from a circular orbit at a rate

$$\left(\frac{\partial \ln J}{\partial t}\right)_{GR} = -\tfrac{32}{5}\,\frac{G^{5/3}}{c^5}\,\frac{M_1 M_2}{(M_1+M_2)^{1/3}}\left(\frac{2\pi}{P}\right)^{8/3}, \tag{2.2.16}$$

where M_1 and M_2 are the masses of the two stars, P is the orbital period, G is the universal gravitational constant, and c is the speed of light. In the absence of mass transfer, two point masses would spiral together from an initial period, P, in a time

$$\tau_{coll} = \tfrac{1}{8}\left|\frac{\partial \ln J}{\partial t}\right|^{-1}_{GR}. \tag{2.2.17}$$

For two stars of solar mass with an initial binary period $P \lesssim 14$ h, this collapse timescale is less than the age of the universe. Indeed, it appears that gravitational radiation is probably the dominant evolutionary process driving cataclysmic binaries in this period range. Gravitational radiation is also no doubt the most important angular momentum loss mechanism in double degenerate binaries, of which the sterling example is the binary pulsar PSR 1913+16, in which the orbital decay attributable to general relativistic effects has actually been measured.

Spin–orbit exchange of angular momentum is potentially of importance in binaries with very extreme mass ratios, in which the rotational angular momentum of one of the two stars is comparable with the orbital angular momentum. The moments of inertia of stars change in the course of their evolution, so that, even in the absence of mass exchange, tides lead to a continuous exchange of rotational and orbital angular momenta in attempting to enforce synchronism. However, if the sum of the moments of inertia of the components is large enough in proportion to the orbital moment,

$$I_1 + I_2 > \tfrac{1}{3}\,\frac{M_1 M_2}{M_1+M_2}\,a^2, \tag{2.2.18}$$

the equilibrium synchronous state is itself secularly unstable, and the two components will either spiral apart or spiral together in the presence of tidal dissipation. This situation may occur for a low-mass component orbiting close to the surface of a massive companion.

Consequential loss mechanisms

A number of conditions arise during the mass transfer process itself which may lead to losses of mass and angular momentum from a binary. One such mechanism was discussed above in the context of accreting stars, namely the loss of orbital angular momentum to rotation of the accreting star. Tides may eventually restore most of that spin angular momentum to the orbit, but this is not the case for most other loss mechanisms.

Some losses no doubt occur from the outer edge of an acfretion disc, and it is likely that mass lost in this way carries off a very high specific angular momentum. On the other hand, unless the mass-losing star is much less massive than the accretor, the fraction of mass transferred which is lost from the system is small. At extremely small mass ratios, tidal action by the low-mass component can no longer extract angular momentum from the disc very efficiently, however, and the absorption of angular momentum by the disc itself can destabilize the binary. Dynamical instability of a low-mass lobe-filling degenerate (or fully convective) star occurs if the mass ratio falls to $q \lesssim \frac{1}{12}$, because expansion of that star's Roche lobe rapidly enough to accommodate the expanding star itself becomes impossible in the face of angular momentum losses to the disc.

The possibility of a *supercritical wind* is raised when mass transfer rates exceed the Eddington limiting rate (equation (2.2.15)). In this case, radiation pressure due to the accretion luminosity can blow the excess mass being stripped from the lobe-filling star out of the system. This is a circumstance which requires enormous (dynamical timescale) transfer rates if the accretor is a normal star, because its gravitational potential well is relatively shallow. But if the accretor is a white dwarf, or more especially if it is a neutron star or a stellar-mass black hole, the mass transfer rates required are much more modest. It is believed that the observed upper limit to the X-ray luminosities of compact galactic X-ray sources at $L_x \sim 3 \times 10^{38}$ erg s^{-1} is due to the onset of a supercritical wind (which would degrade the X-ray flux) at higher accretion rates. And in classical novae, the occurrence of luminosities exceeding the Eddington limit is invariably associated with dynamical mass ejection.

Our discussion would not be complete without noting two other catastrophic mechanisms which are likely to occur during dynamical timescale mass transfer. The first of these is an instability whose existence is implied by the classical Roche potentials described in Chapter 1, namely, the possibility of mass loss from the system through the outer Lagrangian point, L_2. If a star should grow to fill the equipotential surface passing through L_2, a stream could spiral outward from this point. The specific angular momentum per unit mass at L_2 is at least 5.7 times the mean in such a binary, and in fact the stream tends to acquire even more angular momentum through tidal torques as it leaves the system. The result is an extraordinarily rapid decay of the orbit.

Growth of a star beyond its classical outer Lagrangian surface is certainly possible during dynamical timescale mass transfer. In reality, however, it is inconceivable that it could maintain the synchronism assumed by the Roche potentials, and so this highly idealized picture must break down. Nevertheless, the existence of a limit to the extent of a stellar envelope imposed by rotation (as opposed to tides) in a binary system is an important one, and it probably plays an important role in *common-envelope* evolution.

By the term *common-envelope* binaries, we refer here to objects in which two stellar cores have become embedded within a common, more slowly rotating envelope. (For the purposes of the present discussion, we distinguish these objects from the W Ursae Majoris, or contact, binaries, in which a shallow common envelope is maintained in synchronous rotation.) Common-envelope binaries may occur, for example, as a result of the spin–orbit instability discussed above in binaries with extreme mass ratios. Depending upon the direction of instability, the low-mass component can actually spiral into the envelope of its companion before it becomes tidally unstable if its mean density is significantly greater than that of its companion. But common-envelope evolution is probably a much more common occurrence in those binaries which become unstable to dynamical timescale mass exchange. There, the growth of the envelope of the mass-losing star far beyond its Roche lobe and the likely occurrence of supercritical winds and rotational mass loss, in a manner analogous to overflow of the L_2 point, draw the accretor into the envelope of the mass-losing star. Once inside, the two stellar cores experience a gravitational and hydrodynamical drag from the more slowly rotating surrounding envelope. This torque removes orbital energy and angular momentum from the binary core. The dissipation rate of orbital energy (drag luminosity) is probably of order

$$L_D \sim \frac{GM_T a^2}{P} \rho, \qquad (2.2.19)$$

where M_T is the total mass of the two cores, a their separation, P their orbital period, and ρ the ambient density of the surrounding envelope. The two cores will spiral toward each other on a timescale of order

$$\tau_D \sim \frac{\langle\rho\rangle}{\rho} P, \qquad (2.2.20)$$

where $\langle\rho\rangle$ is the mean density of the system within the binary core system. The drag luminosity will tend to inflate the embedding envelope, thereby reducing ρ if the orbital decay timescale, τ_D, is much smaller than the thermal timescale (equation (2.2.4)) of the envelope, τ_{KH}. One therefore expects these two timescales to converge. As we shall see later, evolution of this type appears inescapable if we are to understand the existence of certain types of binaries (most particularly, the cataclysmic binaries), and it is therefore generally assumed that the increase in L_D as the two cores approach each other will suffice to eject altogether the common envelope. An enormous amount remains to be learned about common-envelope evolution, however, not least being how this envelope ejection is triggered physically.

2.2.6 Mass transfer remnants

What determines the point at which mass loss from a lobe-filling star finally ceases? An obvious answer to this question is that it ceases when that star can no longer fill its Roche lobe. In terms of the structure of that star we must look to those phases of evolution which lead to its contraction. From the discussion in Section 2.2.2 above, it is clear that the evolutionary expansion of a star is driven primarily by the gradual chemical differentiation of the star as matter in its central regions is fused to form heavier elements, and by the appearance and growth of a degenerate core when fuels are exhausted there. As a rule, then, expansion terminates with: (1) ignition of a new cycle of nuclear burning (helium or carbon burning) in a degenerate core, which has the effect of powering down the burning shells at the boundary of the core which supply most of the stellar luminosity (if ignition does not blow the star apart altogether); or (2) exhaustion of the envelope of the star which fuels its nuclear energy sources. These conditions coincide with the upper envelope of stellar radii in Figure 2.2.1, and to the degenerate helium ignition limit illustrated there.

The considerations just outlined are not sufficient to fix the terminal state of an interacting binary system, however. One must consider also its orbital evolution. To cast the problem into these terms, we take advantage of the fact that the orbital period, P, of a binary with a lobe-filling component is determined, within a weak function of its mass ratio, $f(q)$, by the mean density, $\bar{\rho}$, of its lobe-filling component:

$$\log P = -0.433 - \tfrac{1}{2}\log\bar{\rho} - \tfrac{3}{2}\log f(q). \qquad (2.2.21)$$

For most mass ratios of interest, the last term is small, $|\frac{3}{2}\log f(q)| \lesssim 0.10$, where the constant in equation (2.2.21) has been chosen so that $f(1) = 1$. We can thus transform Figure 2.2.1 into Figure 2.2.5, which now indicates the critical orbital periods, as a function of the mass of a lobe-filling star, at which that star reaches one of the critical points in its evolution. An evolving primary (the more massive non-degenerate component) in a close binary decreases in mean density (increases in radius) as it evolves, until the critical density fixed by its orbital period is reached. At this point it will become unstable to tidal mass loss. We can thus determine the evolutionary state at which mass transfer begins in a given system as a function of its orbital period and the mass of its primary. (One must of course also take into account spontaneous mass and angular momentum loss, which may alter these parameters between the birth of the binary and its first tidal interaction.)

In the idealized case in which the total mass, $M_1 + M_2$, and total orbital angular momentum, J, of a binary are conserved, a mass-losing star will follow a trajectory in Figure 2.2.5 defined by the varying mass ratio of the binary, since

$$J = [G(M_1 + M_2)^3 a]^{1/2}\mu(1-\mu), \qquad (2.2.22)$$

where a is the orbital separation and $\mu = q/(1+q)$, and the Roche lobe radius, R_L, is a function only of a and q (assuming synchronous rotation). This trajectory is illustrated in Figure 2.2.5 for the lobe-filling component of the well-studied Algol-type binary AS Eridani, which in fact must lie near the termination of mass transfer.

Unfortunately, as outlined in Section 2.2.5, the real world is more complicated than this idealized set of assumptions indicate. Real binaries lose mass and angular momentum from the system, and so the idealized track in Figure 2.2.5 at best only qualitatively reflects reality. Indeed, the trajectory for AS Eri illustrates the point: if we retrace its past evolution assuming it always had its present low angular momentum, we find that when the present lobe-filling star was equal in mass to its companion (as it must have been at some point - it has after all evolved further), the orbital period and binary separation would have been far too small to fit the companion star into the system. The only conclu-

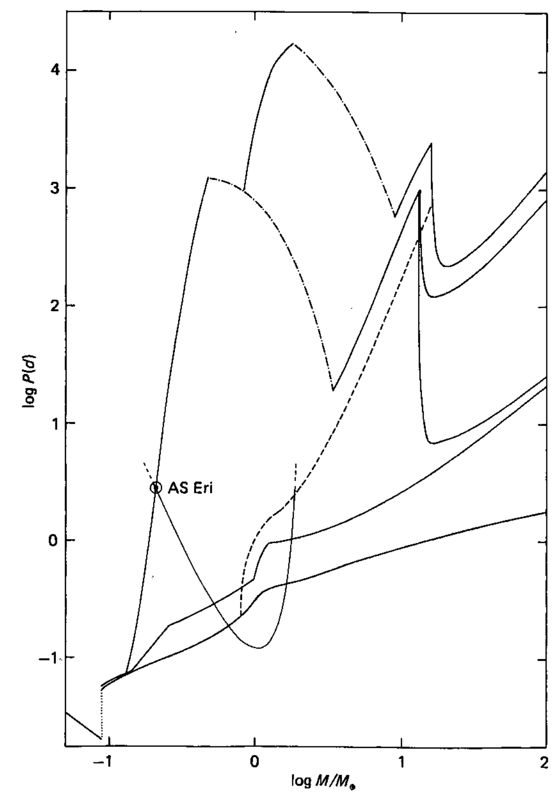


Fig. 2.2.5. Limiting binary periods for the onset of tidal instability at various stages of evolution of the lobe-filling star. The individual curves correspond to the limiting radii of Figure 2.2.1., assuming a binary mass ratio of unity. The curve passing through the point representing the lobe-filling component of AS Eri is the trajectory followed by a system conserving total mass and orbital angular momentum (after R. F. Webbink, 1979, in *IAU Colloquium 53, White Dwarfs and Variable Degenerate Stars*, eds. H. M. Van Horn & V. Weidemann, Rochester: Univ. Rochester Press, p. 426).

sion possible is that AS Eri must have lost a significant fraction (more than one-third) of its orbital angular momentum since the component masses were equal. It is probably accurate to say that, even in the most favorable cases, tidal mass transfer produces remnant binaries of lower angular momentum per unit mass than their progenitors.

Even in the presence of systematic losses of mass and angular momentum, however, several simple statements can be made which constrain possible mass transfer remnants in a given initial binary:

(1) The remnant system cannot possess more mass and angular momentum than its parent.

(2) The remnant cannot possess a smaller core mass than its progenitor: nuclear evolution is irreversible. Some mixing may be possible if an accreting star possesses a burning core, but not if its core is degenerate. Mass-losing components in general preserve their initial composition profiles through rapid mass transfer.

(3) Tidal mass loss ceases from a shell-burning star when the envelope above its outermost active shell has been stripped away. If the binary is unable to stabilize itself against rapid mass transfer (thermal or dynamical timescale), the envelope of the mass-losing star will be stripped away so rapidly that no significant evolution of its core is possible, and its remnant then corresponds to the stripped core of its progenitor at the onset of mass loss. If it can stabilize itself, mass transfer normally proceeds sufficiently slowly to permit the core to grow until the envelope is exhausted by a combination of core growth and mass loss, and the remnant core will be more massive than that of its progenitor.

2.2.7 Statistics of zero-age binaries

We will turn shortly to the matter of actually interpreting known classes of interacting binary systems in terms of their evolutionary status and history, but, before doing so, a few words are in order regarding the distribution in orbital period, mass, and mass ratio of unevolved binary systems. This is an essential ingredient if we are to understand the statistics of evolved binaries, as well as their individual places in close binary evolution.

Ideally,† we would like to be able to specify, if only empirically, the initial distribution function, $\Phi(M, q, P)\,\mathrm{d}\ln M\,\mathrm{d}\ln q\,\mathrm{d}\ln P$, which determines how

† In principle, we should also specify the distribution in initial orbital eccentricity, e. However, under most circumstances, tidal damping can very nearly circularize a binary orbit by the time one component fills its lobe. The binary period of the circularized orbit, P, is then $P = P_0(1-e^2)^{3/2}$, where P_0 is the initial binary period, if the total mass and angular momentum of the binary are conserved. To simplify matters here, we will neglect this added complication.

well-populated different regions in Figure 2.2.5 are. Unfortunately, the determination of Φ is fraught with difficulties because of very strong observational selection effects, which make the detection of certain types of binaries very difficult. However, existing studies of Population I binaries (those in the galactic disc) suggest that Φ is separable – that is, it can be written as a product of distributions in the mass, M, of the more massive (primary) star, ratio q of the mass of that star to its companion, and orbital period, P:

$$\Phi(M, q, P)\,\mathrm{d}\ln M\,\mathrm{d}\ln g\,\mathrm{d}\ln P = f(M)\,\mathrm{d}\ln M\cdot g(q)\,\mathrm{d}\ln q \cdot h(P)\,\mathrm{d}\ln P. \qquad (2.2.23)$$

The distribution of primary masses, $f(M)$, is evidently quite similar to that of isolated single stars. In the mass range above $\sim 0.9\ M_\odot$, that is, among those primaries which evolve beyond the main sequence within the age of the galactic disc, this distribution is well approximated by the Salpeter function,

$$f(M)\,\mathrm{d}\ln M \sim M^{-2.35}\,\mathrm{d}\ln M. \qquad (2.2.24)$$

As for the distribution in mass ratios, $g(q)$, different studies have given conflicting results. The difficulty lies primarily with detecting radial velocity variations due to low-mass companions, determining orbital inclinations, and assuring that companions which are undetectable photometrically or spectroscopically are indeed unevolved, and not mass transfer remnants. Most studies show that $g(q)$ has a maximum at or very near $q = 1$ (its lower limit in the above definition of q), but opinions diverge greatly regarding whether $g(q)$ falls rapidly with increasing q, or indeed whether it may have another maximum near $q = 4$.

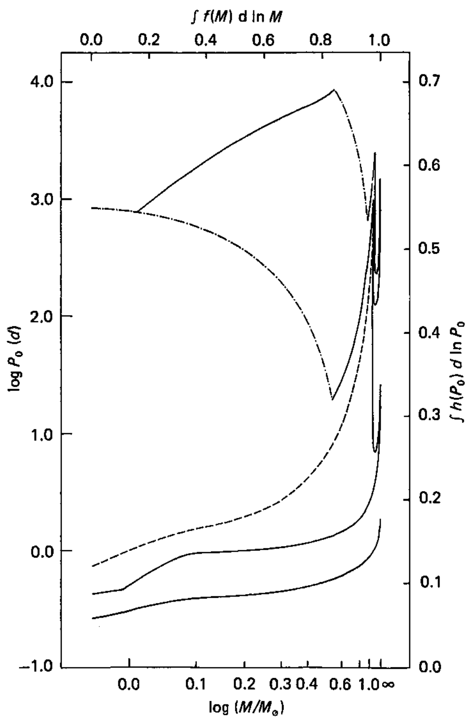
Independent studies of the period distribution, $h(P)$, on the other hand, have generally agreed in finding that the number of binaries per logarithmic interval in period is nearly constant over the range of periods of interest here. This is not a trivial observational deduction, since binaries with periods of several months to a few years tend to have radial velocity amplitudes too small to be obvious, and angular separations too small to be resolved visually. A reasonable representation of $h(P)$ is

$$h(P)\,\mathrm{d}\ln P \cong 0.06\,\mathrm{d}\ln P, \qquad (2.2.25)$$

for periods equaling or exceeding the appropriate minimum as a function of mass in Figure 2.2.5, up to $P \cong 100$ yr.

A useful aid in visualizing the statistical likelihood of binary mass transfer originating at one evolutionary stage or another of the primary star is to map Figure 2.2.5 in such a way that it is uniformly populated by unevolved binaries. The results of this exercise are illustrated in Figure 2.2.6, where we retain the logarithmic scale in period, on which binaries are uniformly distributed, but modify the mass distribution to one of uniform intervals in $f(M)$, rather than in ln M. The lower mass limit, 0.9 $M_\odot$, corresponds to a star of solar composition whose main sequence lifetime equals the age of the galactic disc. If the birthrate of binaries is assumed constant since the creation of the galactic disc, then the probability per unit time that a binary is now first encountering mass transfer is uniform over the permitted period range in this diagram. As we might have

Fig. 2.2.6. The limiting period diagram (Figure 2.2.5.) mapped so that equal areas correspond to equal probabilities of a binary first encountering mass transfer in that phase at the present time. The scale at the top of the diagram marks equal increments in the cumulative mass distribution of the primary component, that at the right equal increments in the cumulative period distribution (normalized to total stellar death rate at each mass). The diagram incorporates the effects of mass loss in a non-magnetic stellar wind during giant branch and asymptotic giant branch evolution.



expected, the most common occurrences of mass exchange at present involve low- and intermediate-mass stars first filling their lobes on the giant branch as hydrogen shell-burning stars, on the asymptotic giant branch as a double-shell-burning stars, or in the Hertzsprung gap, between the main sequence and the giant branch, as stars adjusting to hydrogen shell burning.

2.2.8 Close binary evolution

We now have the ingredients needed to paint a coherent picture of interactive evolution in close binary stars. In this closing section, we will necessarily paint with a broad brush, as it is necessary to omit many details, and many features are still only qualitatively understood. But we can at least hope to perceive things in their broad outline.

First, a word is in order regarding nomenclature. We have found it convenient above to discuss mass transfer as occurring on dynamical, thermal, nuclear, or orbital evolutionary timescales. This is a useful distinction to make, particularly inasmuch as a number of the non-conservative loss mechanisms discussed above appear much more likely to be instrumental in evolution on one of these timescales than on the others. The usual convention in discussing the evolution of a particular binary, however, is to distinguish cases according to the evolutionary state of the lobe-filling star: core-hydrogen burning ('case A'), shell-hydrogen burning ('case B'), or any stage from helium burning onward ('case C'). These labels originated from the earliest theoretical studies of mass transfer. Hybrid cases, where the mass-losing star continues mass transfer through two stages, are often labeled accordingly ('case AB' or 'case BC'), but the notation here does not always conform to the original definitions (the situation in which mass loss is interrupted by core-helium ignition in a red giant often being referred to as 'case BB', for example).

We open the discussion with the *W Ursae Majoris binaries.* The structure of these short-period, low-mass, main sequence contact binaries is discussed in considerably more detail in Chapter 3. The physical contact between their two components opens a possibility which we have not considered above, that of energy transfer between the two stars within their common surface layers, even in the absence of net mass transfer. The observed systems show this phenomenon clearly and, without doubt, it profoundly affects their evolution. With their less massive components inflated by the energy input from their more massive companions, they can accommodate the evolutionary expansion of the more rapidly aging massive component by gradually drawing mass from the smaller star. This they must do if the depth of contact between the stars is not to grow so large that overflow of the outer Lagrangian point occurs, a possibility which can probably only arise if the envelopes of the two stars can find and maintain a stable equilibrium. As noted in Section 2.2.4, their evolution is probably accelerated by angular momentum losses in a stellar wind. Even in the absence of this mechanism, however, their eventual coalescence into a single star is assured by the fact that their total angular momentum is so low that it would be completely absorbed in rotation of the more massive component if that star reached the giant branch. It has in fact been suggested that the *FK Comae stars*, a small class of peculiar, rapidly rotating G and K giants, are immediate descendants of the W Ursae Majoris stars, still in the process of coalescing.

It appears likely that very close binary systems (those transferring mass during core-hydrogen burning of the primary - case A), as a rule, evolve into contact. They have little room in which to accommodate the rapid, non-equilibrium growth of a radiative secondary star as it accretes matter, and such a process has in fact been suggested for the origin of the W Ursae Majoris systems. More massive, *early-type contact binaries* also show evidence of energy exchange between their components, and are likely to have originated in this way. They may very well parallel the W Ursae Majoris systems in long-term evolution towards coalescence, but very little theoretical work has been done so far on their structure or evolution.

Systems which first reach mass transfer while the more massive component is evolving across the Hertzsprung gap (orbital periods between the terminal main sequence and the base of the giant branch in Figure 2.2.5) have radiative envelopes and therefore transfer mass on a thermal timescale if rapid mass transfer cannot be averted. Even so, in these circumstances the mass transfer rates are sufficiently moderate that the most devastating mass and angular momentum loss mechanisms can probably be avoided. Similarly, at the somewhat longer orbital periods involved, binaries stand a better chance than in case A transfer of avoiding evolution into contact, at least if the component masses are not very disparate. We may therefore expect these systems to evolve quasi-conservatively, that is, in a manner qualitatively similar to that described by the most straightforward assumptions of constant total mass and orbital angular momentum.

The *Algol-type binaries*, whose interpretation in terms of large-scale mass transfer stimulated the first theoretical studies of close binary evolution, are clearly products of quasiconservative mass transfer. It was the success of detailed calculations in producing objects of this type assuming strictly conservative mass transfer which marked the earliest and most striking successes of close binary evolutionary theory, even though it is now clear that Algol systems actually suffer appreciable angular momentum losses - AS Eri discussed in Section 2.2.6 above is a case in point.

The great majority of Algol binaries have total masses less than $\sim 7\ M_\odot$, a feature which is only partly accountable in terms of the larger birthrate

of low-mass stars, since more massive systems are also visible at much greater distances. It probably also reflects the fact that original primary components of mass $\lesssim 3.5\ M_\odot$ reaching mass transfer in the Hertzsprung gap have small enough cores to permit a prolonged phase of nuclear timescale mass transfer following reversal of the binary mass ratio, as the mass-losing star evolves up the giant branch. They ultimately die as very low-mass helium white dwarfs. This is not true of their more massive counterparts, in which the whole of the primary's envelope is lost on a thermal timescale. In the more massive systems, the remnant of the lobe-filling star can ignite helium, continuing its life as a helium star. A few examples of this type are known, such as KS Persei and υ Sagittarii. If the helium stars in these binaries are massive enough, they may grow to encounter a second phase of mass loss (case BB), stripping the star down to its carbon-oxygen core, and leaving a massive ($\sim 1.0\ M_\odot$) white dwarf remnant.

In very massive stars ($M_1 \gtrsim 12$–$14\ M_\odot$), core-helium ignition follows close on the heels of hydrogen exhaustion while these stars are still in the Hertzsprung gap (see Figure 2.2.1). When stripped of their hydrogen envelopes by a companion, these stars expose a very luminous, massive ($\gtrsim 4\ M_\odot$) helium-burning core, which is probably observable as a *Wolf-Rayet star*. At one time it was thought that a large fraction of Wolf-Rayet stars were produced in this way by tidal mass transfer. But with the growing appreciation of the magnitude of stellar wind mass loss in massive single stars, the role of the companion star in exposing the helium core has become clouded, even though the rate of duplicity among Wolf-Rayet stars is quite high.

The helium star in these Wolf-Rayet systems is massive enough to ignite carbon non-degenerately, and may therefore be expected to produce a neutron star remnant in a supernovae event, or, in the most massive cases, a black hole. Since this star is much less massive than its companion when the supernova occurs, the binary stands an excellent chance of remaining bound. The *massive X-ray binaries* are believed to be remnant systems of such an event, now on the verge of entering the reverse phase of mass transfer in which the originally less massive star has evolved to the point of nearly filling its Roche lobe. Tidal overflow would produce thermal timescale mass transfer rates far greater than the Eddington-limited accretion rate for a neutron star. Since the excess mass flux would quickly extinguish the X-ray flux from the accretor, it is more likely that in most observed massive X-ray binaries, accretion from the stellar wind of the massive component is powering the neutron star X-ray source.

A widely accepted model for the creation of *binary radio pulsars* is that they are produced when the massive components in these massive X-ray binaries follow a course of evolution and mass transfer parallel to that of the original primary, and terminating in a second supernova event in the same binary. Under most circumstances, the mass ejected by this supernova is so great that the binary would be disrupted, but in a few cases a double neutron star will survive in a very eccentric orbit. Four binary radio pulsars are now known, and the first of these to be discovered (PSR 1913+16) indeed has a very eccentric orbit ($e = 0.62$) and a short orbital period. However, the three examples found since (PSR 0820+20, PSR 0655+64, and PSR 1953+29) all appear to have nearly circular orbits, in two cases with very long periods, a circumstance which is virtually irreconcilable with this model for their origin.

Turning now to those binaries of such long period that they only encounter mass transfer after the more massive star has reached the giant branch, we are faced with a class of objects whose first phase of mass transfer must be extremely intense and very short-lived by any evolutionary standard. Not only are the thermal and nuclear timescales of giants and asymptotic branch giants short, but they possess the deep convective envelopes which tend to be unstable to dynamical timescale mass loss when they fill their Roche lobes.

At the low-mass, short-period extreme of this range of masses and orbital periods we find the *RS Canum Venaticorum systems.* These are binaries consisting typically of a giant of G or K spectral type with an F- or G-type main sequence companion, and they display a variety of photometric and spectroscopic peculiarities. They are not yet actually in the process of tidal mass transfer, but the great majority of them clearly lie near this first phase of mass transfer.

At very long periods, the dynamical timescale of a lobe-filling star is longest, and we therefore stand the best chance of catching a binary at the onset of this intensely interactive phase. Many *symbiotic stars* have orbital periods and masses in this range, and, for the few systems with spectroscopic mass ratios, it is found that the cool giants in these binaries are the more massive components. It has been suggested that some of them (such as T Coronae Borealis and CI Cygni) are in the throes of the first phases of dynamical timescale mass transfer, in which the hot subdwarf spectral component is actually produced by an accretion disc through which mass is flowing at $\sim 10^{-5}\ M_\odot\ yr^{-1}$ onto a main sequence star. Their outbursts are attributed to sudden increases in the mass flux through the disc. Other symbiotic stars (such as AG Pegasi and RW Hydrae) appear to contain hot degenerate companions, accreting material from the stellar winds of their giant companions. In some cases, very slow nova-like outbursts have been observed in these systems.

Because of the catastrophic mass and angular momentum loss mechanisms which are probably activated by dynamical timescale mass transfer, the remnant binaries of these systems (if they survive at all) are probably to be found among very short-period binaries containing a white dwarf component – the stripped core of the original giant – and a main sequence companion. This is believed to be the process responsible for the creation of the very short-period *planetary nebulae with close binary nuclei*, and for the origin of the *cataclysmic binaries*, both of which contain degenerate dwarfs with low-mass main sequence companions – presumably the stripped cores of red giants or asymptotic branch giants and their companions which spiraled in through the common envelope.

With few exceptions, the cataclysmic binaries have such short orbital periods that their main sequence components cannot be massive enough to drive mass transfer at the observed rates through nuclear evolution alone. Evidently, it is driven by a combination of gravitational radiation and magnetically coupled stellar winds, both of which sap angular momentum from the system. Their long-term evolution is complicated by the repeated occurrence of thermonuclear outbursts (novae) as hydrogen is deposited onto the degenerate white dwarf star. These outbursts are capable of ejecting a large fraction of the matter accreted in the interval between them, and if material from the degenerate core itself is mixed into the accreted envelope by shear turbulence, it is possible that the white dwarf, as well as the red dwarf, loses mass with time. The large abundances of carbon, nitrogen, and oxygen observed in the ejecta of many novae suggests that this is the case. Nevertheless, it has been widely speculated that sufficiently massive white dwarfs in cataclysmic binaries might be pushed over the Chandrasekhar limit by mass accretion from their companions and produce type I supernovae. In other systems, it appears possible that in the long run the main sequence star can be eroded to such a small mass that the gravitational radiation timescale for the binary becomes shorter than the thermal timescale of the main sequence star. At this point, the mass-losing star (which by now is fully convective) ceases to contract (see ζ_s in Figure 2.2.3), but begins cooling to degeneracy, and the orbital period reaches a minimum observed to be ~80 min.

The most massive red supergiants contain cores so massive that they are capable of igniting carbon non-degenerately, and thus surviving to produce neutron stars. The suggestion has been made that the *low-mass X-ray binaries* were produced in binaries containing such supergiants with low-mass companions, through the same sort of common-envelope evolution as is believed to produce the cataclysmic binaries. The evolution of these low-mass X-ray binaries no doubt parallels that of the cataclysmics, though without the nova outbursts. At the low-mass transfer rates prevalent in these systems, type I X-ray bursts are produced by repeated ignition of the helium shell on the neutron star, but these are energetically incapable of ejecting much of the accreted mass. It is possible, then, for accretion from the main sequence star ultimately to push the neutron star over its upper mass limit to form a black hole. No obvious candidates for low-mass systems containing black holes have yet been identified, however.

We close the discussion of close binary evolution by returning to the Algol binaries. When we left them last, they had evolved to a state in which the originally more massive star had been reduced to a white dwarf orbiting a now much more massive main sequence companion. It appears likely that such systems, when they again encounter mass transfer because of evolution of the remaining star, will enter a common envelope phase of evolution not unlike that discussed above. This fate is favored both by their extreme mass ratios, and by the fact that the main sequence star will generally reach the giant branch before filling its lobe in these Algol remnants. The product of such a phase will not be a conventional cataclysmic binary, but a *close double white dwarf* system – the remnant cores of the two stars. Three such objects are known – AM Canum Venaticorum, PG 1346+082 and GP Comae. All of these are helium-rich objects, themselves apparently undergoing mass transfer. Their orbital periods (up to 46 min) are in fact remarkably long for interacting white dwarfs, and indicate that the lower-mass components of these binaries have just about been completely devoured. It is not unlikely that there exist more massive counterparts to these systems, ones in which the total mass of the two white dwarfs exceeds the Chandrasekhar limit for white dwarf masses. Their denouement may well be that, after waiting, literally for eons, for gravitational radiation to draw them together, they coalesce on a dynamical timescale, and, with their combined mass, evolve into type I supernovae.

2.3

X-RAY BINARIES: END POINTS OF BINARY EVOLUTION

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2.3.1 General considerations

In this section we discuss in more detail the X-ray binary stars mentioned above. These are objects which emit the majority of their radiation in the X-ray range of the spectrum (0.1-100 keV). They are powered by matter being transferred from a companion star onto a compact object – a neutron star or a black hole. Similar considerations apply when the accreting object is a white dwarf (see Chapter 4). The depth of the potential well of an object of mass M and radius R is GM/R and measures the efficiency (in ergs per gram) with which matter falling onto it can be converted to radiation. For a neutron star, typical parameters are $M = 1\ M_\odot$ and $R = 10^6$ cm, and so one gram falling onto it releases about 10^{20} erg, equivalent to 10 per cent of the rest mass energy of the accreted material. Similarly, up to around 30 per cent of the rest mass energy of matter accreting onto a black hole ('Schwarzschild' radius $R_S = 2GM/c^2 = 3 \times 10^5\ (M/M_\odot)$ cm) can be released as radiation. The velocity reached by matter falling radially inwards – the free-fall velocity v_{ff} – is given by $\frac{1}{2}v_{ff}^2 = GM/R$. For a neutron star $v_{ff} \cong 0.3c$, where $c = 3 \times 10^{10}$ cm s^{-1} is the velocity of light. By definition, for a black hole at the Schwarzschild radius, $v_{ff} = c$. The efficiency with which matter is converted into radiation by accretion onto compact objects far exceeds the fraction of 1 per cent which is available from nuclear fusion. Typical X-ray luminosities in these objects range up to 10^{38} erg s^{-1}, comparable to the optical luminosities of the brightest stars, and to achieve this an accretion rate of only 10^{18} g s^{-1} or $10^{-8}\ M_\odot$ yr^{-1} is required. The minimum temperature at which a body of radius R can radiate a luminosity L is just the black body temperature T_{bb}, where

$$L = 4\pi R^2 \cdot \sigma T_{bb}^4$$

and σ is the Stefan-Boltzmann constant. For a neutron star of radius 10^6 cm and luminosity 10^{38} erg s^{-1} the temperature required is 2.10^7 K. Such emission peaks at a frequency of 1.2×10^{18} Hz, corresponding to a wavelength of 2.5 Å