

Nature of stars:

When WD's mass exceeds M_{\odot} , it begins to collapse and $\rho \tau$. At some point, the e^- Fermi energy $E_F = \sqrt{p_F^2 + m_e^2}$ exceeds $(m_n - m_p)c^2 = 129$ MeV. This happens at density $\rho \approx 1.2 \times 10^7 \text{ g/cm}^3$. At higher ρ , it becomes energetically favorable for e^- 's to inverse-beta decay (combine) with protons to form neutrons \Rightarrow neutron star.

More realistically, WD contains CO, not p free protons. What really happens is that e^- 's capture on nuclei to form increasingly n -rich nuclei. At density $\rho \gtrsim 4 \times 10^{10} \text{ g/cm}^3$, neutrons begin to bleed from nuclei. This "ionization" of n 's from nuclei softens the EOS, and star continues to collapse until $\rho \gtrsim 10^{14} \text{ g/cm}^3$ at which point neutron degeneracy pressure becomes sufficient to support the star.

The density $\rho_{\text{nuc}} \approx 2 \times 10^{14} \text{ g/cm}^3$ is the density of ordinary nuclear matter. The EOS at higher densities is determined by nuclear theory, which is highly uncertain.

More quantitatively, there will be degenerate n , p , and e^- with Fermi energies $E_F(n)$, $E_F(p)$, $E_F(e)$. If $E_F(n) < E_F(p) + E_F(e)$, then neutrons cannot β decay. In thermal equilibrium, $E_F(n) = E_F(p) + E_F(e)$, and $p_F = (3n/8\pi)^{1/3} h$ for all. At ρ_{nuc} ,

$$E_F(n) \approx m_n c^2 + \frac{[E_F(n)]^2}{2m_n} \quad E_F(p) \approx m_p c^2 + \frac{[E_F(p)]^2}{2m_p},$$

$$\text{but } E_F(e) \approx p_F(e)c.$$

Using $n_e = n_p$, we find

$$\left(\frac{3n_p}{8\pi}\right)^{1/3} h c + \left(\frac{3n_p}{8\pi}\right)^{2/3} \frac{h^2}{2m_p} - \left(\frac{3n_h}{8\pi}\right)^{2/3} \frac{h^2}{2m_n} \approx (m_n - m_p)c^2 \\ = 1.3 \text{ MeV}$$

Can then solve this at any density noting that

$$(n_n + n_p) = P/m_p.$$

E.g., at ρ_{nuc} $(n_e/n_n) \approx \frac{1}{200} \Rightarrow$ neutron star!

Mass Radius relation for NSs

The WD central density relation,

$$\rho_c = \frac{31}{4\pi^2} \left(\frac{M}{M_\odot} \right)^2 \frac{m_p}{(cmec)^3} \frac{(B)}{cm}$$

is easily modified for NSs by $\gamma_e = 1$, $m_p \rightarrow m_n$.

Likewise, the radius is (roughly)

$$R = \frac{\beta^{1/3}}{2} 0.77 \gamma_e^{5/3} \left(\frac{M_n}{M} \right)^{1/3} \frac{h}{G m_n c}$$

γ_e
relative
to WD

$$\simeq (0.0135) 2^{5/3} \frac{m_p}{m_n} \left(\frac{M_\odot}{M} \right)^{1/3} \simeq 2.13 \text{ km}$$

$$\simeq 2.1 \times 10^{-5} R_\odot \left(\frac{M}{1.4 M_\odot} \right)^{1/3} = 1.5 \times 10^6 \text{ cm} \left(\frac{M}{1.4 M_\odot} \right)^{1/3}$$

$$\simeq 15 \text{ km} \left(\frac{M}{1.4 M_\odot} \right)^{1/3}$$

A few comments/caveats:

- ① As will be seen below, the (Schwarzschild) radius of a $1.4 M_\odot$ BH is $R_{sh} \simeq 3 \text{ km}$. Therefore, the NS is in the strong-field regime of GR. Alternatively, the redshift from the surface of the neutron star is

$$\frac{GM}{c^2} \frac{\Delta \lambda}{\lambda} \simeq 0.2 \%$$

Thus, GR corrections to the Newtonian eqn. of hydrostatic equilibrium will give rise to $\mathcal{O}(20\%)$ correction.

This GR eqn. of hydrostatic equilibrium is known as the Tolman-Oppenheimer-Volkoff (TOV) zero equation.

② The approximation $P \propto \rho^2$ used to derive $P \propto R^{-2}$ breaks down already for $M = 1.4 M_\odot$; in fact $E_F \approx m_e c^2$ there and so a more accurate EOS must be used.

③ In our simple calculation, $P_c \approx 3.3 \times 10^{17} \text{ g/cm}^3 \geq P_{\text{nucl}}$.

Uncertainties in the nuclear EOS at these high densities give rise to additional uncertainties in the NS M-R reln.

E.g., nucleon-nucleon repulsion might stiffen EOS. But if new particles (e.g., π^0 's, K^0 's, hyperons) are produced at high P , this might soften the EOS.

④ The gravitational binding energy/c² is

$$\frac{E_B}{c^2} = \frac{GM^2}{R} \approx 0.1 (1.4 M_\odot).$$

Thus, the mass of a NS is smaller than might expect from Newtonian calculation.

Maximum Mass for NSs?

Incidentally, people now try to determine the nuclear EOS by measuring the M-R reln for NSs. E.g., GM/R can be obtained from redshifts of lines emitted from the NS surface. The surface gravity GM/R^2 can be measured through its effects on pressure broadening of lines. A combined measurement of M/R and M/R^2 can be used to obtain M and R . Are difficult but ambitious people are trying.

Machon Mass for WDs:

Nearly scaling the WD result, we obtain

$$M_{\text{ch}} \approx 6 M_{\odot} \quad \text{for NS}$$

However, all complications above are important, and generally tend (especially GR) to reduce M_{ch} . State-of-the-art calculations for different EOSs give $M_{\text{ch}} \approx 1.5-3 M_{\odot}$.

For fun, suppose we had a star consisting of incompressible matter of density ρ_0 . In Newtonian gravity,

$$\frac{dP}{dr} = -\frac{Gm r P}{r^2} \Rightarrow P(r) = \frac{2\pi}{3} G (R^2 - r^2), \text{ and}$$

$$P_c \equiv P(r=0) = \frac{2\pi}{3} G \rho_0^3 R^2 = \left(\frac{\pi}{6}\right)^{1/3} G M^{2/3} \rho_0^{4/3}.$$

In GR, though the TOV eqn is

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \frac{(1+P/\rho c^2)(1+4\pi r^3 P/mc^2)}{(1-2Gm/r c^2)}$$

which integrates to

$$P = P_0 c^2 \left[\text{something complicated} \right]$$

and

$$P_c = P_0 c^2 \left[\frac{1 - (1-2GM/r c^2)^{1/2}}{3(1-2GM/r c^2)^{1/2} - 1} \right].$$

Then, have $P_c < \infty$ only if $\frac{GM}{R c^2} < \frac{4}{9}$,

which is a slightly stronger bound than $\frac{M}{R c^2} < 2$ from the Schwarzschild radius.

For constant density ρ_0 , this yields

$$M < \frac{8}{27} \left(\frac{c^2}{G}\right)^{3/2} \left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$$= 7.5 \left(\frac{\rho_0}{\rho_{\text{nuc}}}\right)^{1/2} M_{\odot}$$

A stiff enough bond to M/R is obtained by considering causality, which requires $C_S^2 = (\partial P/\partial \rho) < C^2$, or $P = \rho c^2$. This results in $\frac{GM}{Rc^2} < 2291 \frac{1}{2.9}$

Pulsar spin-down:

Pulsars are rapidly rotating NSs that can produce pulsating signals with periods \sim msec - secs. E.g., the Crab pulsar in the Crab nebula from a SN in 1054 AD. It has period $P = 33$ ms and is slowing down with

$$\frac{dP}{dt} = \frac{ms}{90\text{ yrs}}$$

The rapidity of the pulses identifies pulsars as NSs. A star has a max spin freq ω_{max} from

$$\frac{GM}{R^2} = R\omega_{max}^2 \Rightarrow P_{min} = \frac{2\pi}{\omega_{max}} = 2\pi \left(\frac{R^3}{GM}\right)^{1/2} \sim t_{ff}$$

$$\text{or } P_{min} = 10^4 \sqrt{\frac{(R/R_0)^{3/2}}{(M/M_0)^{1/2}}} \text{ sec}$$

$$\text{for } M \sim M_0 \quad P \sim \frac{33\text{ msec}}{R^{2/3}(10^{10})^{2/3} R_0} \stackrel{7}{\approx} \stackrel{100}{\approx} 10 \text{ cm to km}$$

If we approximate NS by uniform-density sphere, then the moment of inertia is

$$I = \frac{2}{5}MR^2 = 2.5 \times 10^{45} \left(\frac{M}{1.4M_0}\right) \left(\frac{R}{1.5 \times 10^6 \text{ cm}}\right)^2 \text{ g-cm}^2$$

The Crab pulsar slows down at a rate

$$\frac{d\omega}{dt} = -2.4 \times 10^{-9} \text{ sec}^{-2}$$

implying a spin-energy loss rate

$$\frac{dE_{int}}{dt} = I\omega \frac{d\omega}{dt} \simeq 4.6 \times 10^{38} \frac{\text{erg}}{\text{sec}}$$

which is comparable to the Crab-nebula luminosity, $5 \times 10^{38} \frac{\text{erg}}{\text{sec}}$

The general equation for the energy loss due to dipole radiation if the dipole axis (the Earth) is tilted at an angle θ to the rotation axis by an angle ϕ , it radiates with

$$\frac{dE}{dt} = \frac{2}{3C^3} m^2 w^4 \sin^2 \phi \quad \left(= \frac{2}{3C^2} |\vec{\omega}_B|^2 \right),$$

where $m \approx B_0^2 R^3$ is the magnetic dipole which is required for Crab to be main $\approx \cancel{3 \times 10^{20}} - 6 \text{ cm}^3$
~~and~~ $B_0 \approx m \sin \theta \approx 4 \times 10^{30} \text{ G-cm}^3$ or

$$B \approx 10^{12} \text{ G.}$$

This seems large, but m_{dipole} can be enhanced by large factor if initially magnetized pre-collapse iron core contains the B field as it collapses.

If B-dipole radiation is at work in the Crab, then

$$\frac{dE}{dt} = Iw \frac{dw}{dt} \propto w^4 \quad \text{or} \quad \frac{dw}{dt} = -Cw^3$$

where $C = 3.5 \times 10^{-16} \text{ sec}^{-1}$ for $w = 1905^{-1}$ $\frac{dw}{dt} = -2.4 \times 10^{-9} \text{ s}^{-2}$.
 Integrating,

$$t = \frac{1}{2C} \left[\frac{1}{w^2} - \frac{1}{w_i^2} \right] < \frac{1}{2Cw^2} = 1253 \text{ years},$$

$w_i < \infty$

as opposed to historical age (≈ 950 yrs) since SN.

The braking index is defined to be

$$n \equiv \frac{-w \ddot{w}}{\dot{w}^2}.$$

It is $n=3$ for magnetic-dipole model and, e.g., 5 for emission of GWs. Measured to be 2.515 for Crab and 2.83 for PSR1509-58.

Ansatz:

Suppose the NS has a \vec{B} -dipole aligned with spin axis.

$$\vec{B} = B_p R^2 \left(\frac{\cos \theta}{r^3} \hat{e}_r + \frac{\sin \theta}{2r^3} \hat{e}_\theta \right).$$

Inside NS, there are enough free e^- 's so that MHD approximation holds; i.e., the e^- 's rearrange themselves so that they are not accelerated—they short out the fields. I.e.,

$$\vec{E}^{(in)} + \frac{\vec{\Omega} \times \vec{F}}{c} \times \vec{B}^{(in)} = 0.$$

Thus, just inside the surface, $\vec{E}^{(in)} = \frac{R \Omega B_p \sin \theta}{c} \left(\frac{\sin \theta}{2} \hat{e}_r - \frac{\cos \theta}{2} \hat{e}_\theta \right)$

The \hat{r} component of \vec{E} ~~at just near the star~~ may jump at the NS surface if charges accumulate there, but the tangential component is const. Thus, ^{just} outside the star,

$$E_\theta^{(out)} = -\frac{\partial}{\partial \theta} \left(\frac{R \Omega B_p \sin^2 \theta}{2c} \right) = \frac{\partial}{\partial \theta} \left[\frac{R \Omega B_p}{3c} P_2(\cos \theta) \right].$$

Outside the star, $\vec{E}^{(out)} = -\vec{\nabla} \phi$ with $\nabla^2 \phi = 0$, implying

$$\phi = -\frac{\sqrt{B_p}}{3c} \frac{R^5}{r^3} P_2(\cos \theta) \text{ to satisfy BC.}$$

i.e., a quadrupole electric field is induced.

Inside the star, there is a charge density $\rho_e = \frac{1}{4\pi} \vec{\nabla} \cdot \vec{E} = -\frac{1}{2\pi c} \vec{\Omega} \cdot \vec{B}$

$$\text{or } \rho_e = 7 \times 10^{-2} B_p \text{ P cm}^{-3}.$$

At the surface of the star, there is a surface charge density $\sigma = \frac{1}{4\pi} (\vec{E}_{out} - \vec{E}_{in})_r = -B_p \Omega R \sin^2 \theta / 4\pi c.$

Inside, $\vec{E} \cdot \vec{B} = 0$, but outside (assuming vacuum outside),

$$\vec{E} \cdot \vec{B} = -\frac{R \Omega}{c} \left(\frac{R}{r} \right)^7 B_p^2 \cos^3 \theta.$$

THUS,

$$E_p \sim \frac{B^2}{c} R_p - 2\pi R_p B_p \Omega_{\text{NS}}$$

This is much larger than particle energy for protons,

$$\frac{\text{electric}}{\text{gravitational}} \sim \frac{e R_p B_p c}{G M_{\text{NS}} / R^2} \sim 10^5 \gg 1$$

and is $\sim 10^{12}$ for e^- 's.

\rightarrow Vacuum is unstable!

\rightarrow Particles are stripped from surface of NS, and NS is necessarily surrounded by plasma of charged particles tied to \vec{B} -field lines; magnetosphere.

Resulting picture is that \vec{B} -field and plasma corotate with NS (like an egg beater) out to light cylinder of radius $R_c = c/\Omega = 5 \times 10^9 \text{ cm}$ beyond which co-rotation would imply $V > c$ for plasma. What happens at larger radii is unclear, but probably involves bending back of field lines to toroidal component, and perhaps expulsion of highly relativistic particles.

Within light cylinder $\vec{E} \cdot \vec{B} = 0$, and $\vec{E} = -\left(\frac{\Omega \times \vec{r}}{c}\right) \times \vec{B}$.

Thus $\vec{E} \times \vec{B}$ with $|\vec{E}| \approx |\vec{B}|$ at light cylinder, thus driving Pointing flux

$$S \sim c \frac{\vec{E} \times \vec{B}}{4\pi} \sim \frac{c B^2}{4\pi} \quad \text{over area } \sim \pi R_c^2, \text{ or}$$

luminosity,

$$\begin{aligned} \frac{dE}{dt} &\sim B_c^2 R_c^2 c \sim c \left(\frac{B_p R^3}{R_c^3} \right)^2 R_c^2 \\ &\sim \frac{B_p^2 R^4 \Omega^4}{c^3} \end{aligned}$$

Therefore, get same spindown from pulsar magnetosphere as in more magnetic-dipole model, even if ~~$B_p \ll B_c$~~ $B_p \parallel \vec{B}_c$.

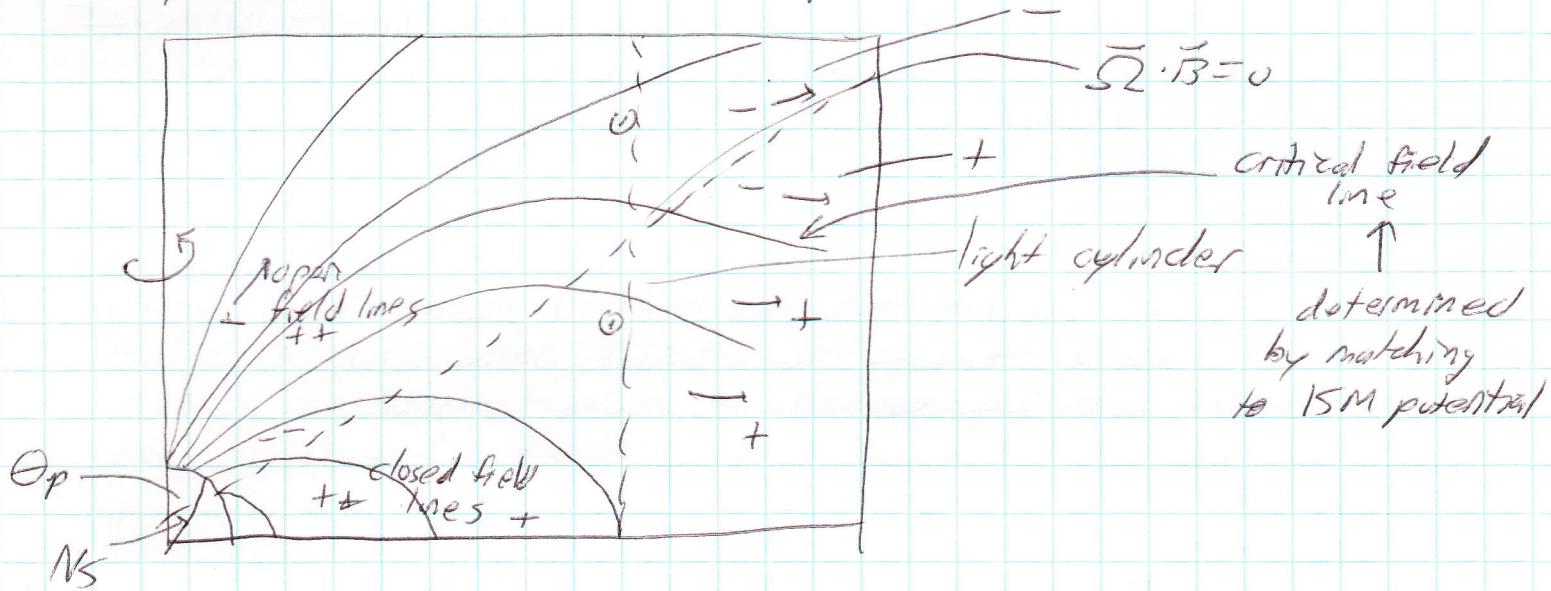
$B = \frac{C}{r^3} \propto r^{-3}$ — very flat.

$$\Rightarrow B \sim \frac{10^4 G}{(pc)} \text{ for } r > R_c \Rightarrow B \sim 10^4 G @ pc \text{ as seen}$$

If particle energy $\epsilon_p \gg \frac{B^2}{8\pi}$, then $B \propto 1/r$,

$$B \sim 10^{-14} G @ r \approx pc; \text{ too small.}$$

The picture of the pulsar magnetosphere is:



Angular size of polar cap is Θ_p , determined by noting that $\sin^2 \theta / R$ is const along the dipole field lines. Thus, the $\Theta_p \sim \sqrt{2R/C}$, and the polar cap has area

$$A_p \sim \pi R^2 \Theta_p^2 \sim \pi \frac{2R^3}{C}$$

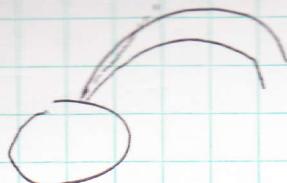
Pulsar emission mechanisms:

In spite of huge amount of detailed data, these are not well understood. In part this is because pulsed radio emission is "dope" — constitutes $\lesssim 10^{-4}$ (e.g. in mJy) of total emission.

Pulsar with $\dot{E} \gtrsim 10^{38}$ erg s $^{-1}$ and pulsation period $\lesssim 10^2$ s, magnetic field $B \lesssim 10^5$ G

Implies $T_b \sim 10^5 - 10^6$ K $10^7 - 10^{12}$ eV

\Rightarrow not synchrotron emission



One fairly universal ingredient:

Particles accelerated ~~shortly~~ by E field will ~~cross~~ become very energetic. E.g., ~~in polar cap, the voltage is~~ potential drop between pole and equator in pulsar is

$$V \sim V \sim \frac{R \Omega}{c} B \Omega \sim 10^{16} \text{ V} \quad \text{for } \Omega \sim 10^4 \text{ Hz}, B \sim 10^{12} \text{ G}$$

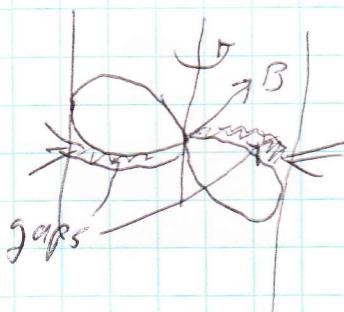
(\gg MeV/e)

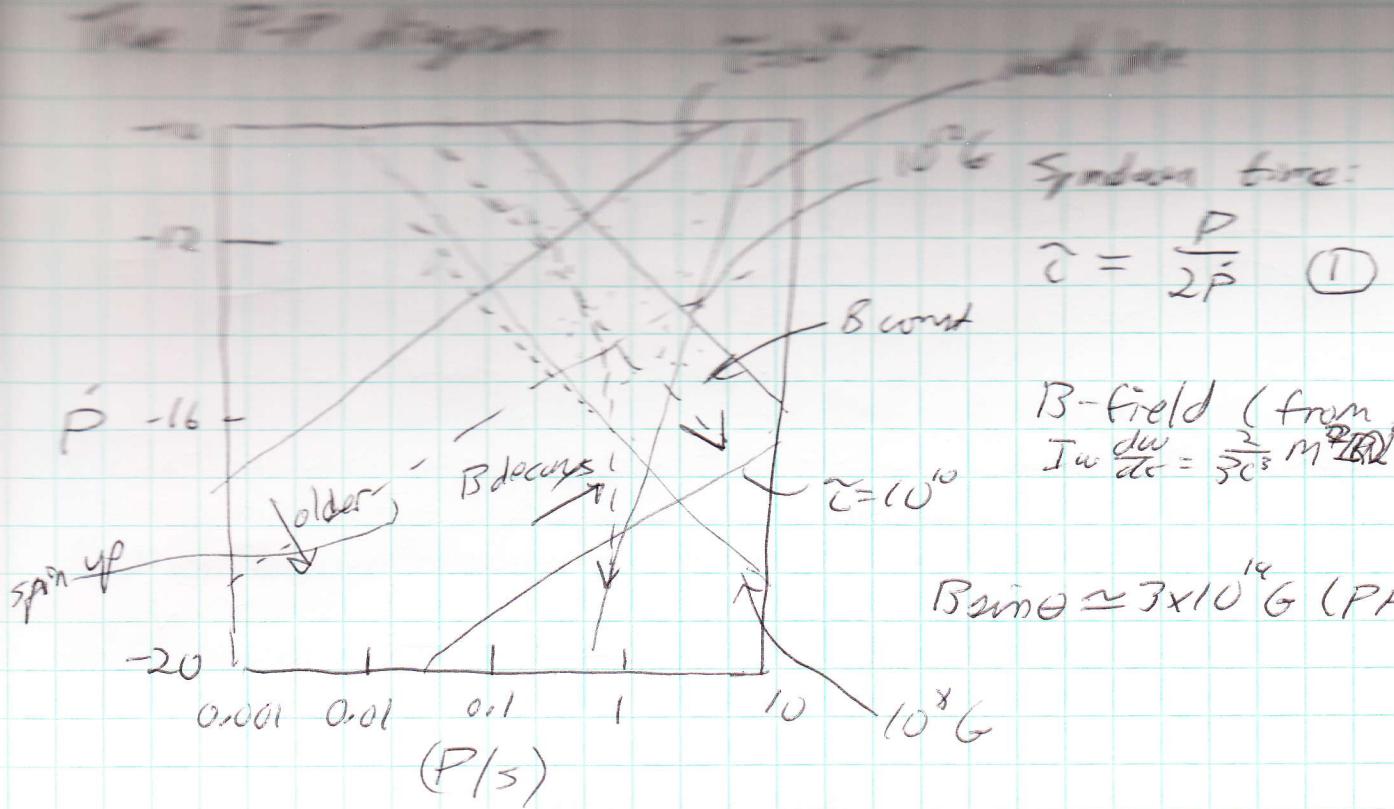
Thus e^- 's attain $\gamma \gg 1$. They then produce curvature radiation as they cross B -field lines. These γ -rays then escape (high-energy emission in polar-cap models) and/or run into other γ 's (or B field) to produce e^+e^- pairs. The magnetosphere is therefore populated by a pair plasma.

There may also be large-scale charge separations; this may then allow for coherent emission (where intensity scales as N^2 , rather than N) which may explain $T_b \gg 10^5$ K.

In polar-cap models, all this takes place near poles, and emission is along axis of symmetry.

In outer-gap models (or light-cone models), emission occurs near light cone near equator in NSs with misaligned B fields.





(3) Evolutionary tracks

(4) Death line: Voltage drop $\propto B \Omega^2 \propto B P^{-2}$ (typical Langer?)

expect e^+e^- pairs to be produced only for $B P^{-2} \gtrsim 10^{11} G s^{-2}$; or $P \propto \Omega^2$ gives line $P \propto P^3$ toward right. Expect all pulsars to be near left.

(5) Spin-up limit. Pulsars in binaries do not follow same rules nor occupy same regions of $P\text{-}\dot{P}$ space.

May be spun up by accretion from companion. Are expected to have $P \gtrsim 2(B/10^9 G)^{6/7}$ ms.

Properties of Neutron Stars

Matter, Forces

$$E_{grav} \sim \frac{GM^2}{R} \sim 5 \times 10^{50} \text{ erg} \sim \frac{1}{5} M_p c^2$$

$$g \sim \frac{GM}{R^2} \sim 2 \times 10^{12} \frac{\text{cm}}{\text{sec}^2}$$

$$\bar{\rho} \sim \frac{3M}{4\pi R^3} \simeq 7 \times 10^{14} \text{ g/cm}^3 \simeq 2-3 \times \text{normal nuclear density.}$$

Structure: atmosphere/outer crust / inner crust/outer core / inner core

Atmosphere: $\sim 10 \text{ cm}$ for $T_s \sim 3 \times 10^6 \text{ K}$

to $\sim 2 \text{ mm}$ for $T_s \sim 3 \times 10^5 \text{ K}$

models not complete for $B \gtrsim 10^{10} \text{ G}$ or $T_s \lesssim 10^5 \text{ K}$

$\rho \lesssim 10^6 \text{ g/cm}^3$; negligible mass; primarily ^{56}Fe

shapes emergent spectrum

plasma layer; highly ionized; atomic levels distorted by high B fields

Outer crust: ("outer envelope")

from atmosphere to $\rho \sim 4 \times 10^{11} \text{ g/cm}^3$

100s m thick

e^- 's + e^- 's (magnetic effects may be important).

top few m's is nondegenerate e^- gas

deeper \rightarrow degenerate e^- gas, ultrarelativistic

solidifies at high depth

β capture in nuclei $\rightarrow n$ -rich nuclei

bottom of crust: n 's start to drop, form n gas
($\rho \sim 4 \times 10^{10} \text{ g}$).

n gas is still background sea for nuclei

inner crust: $\sim 1 \text{ km}$ thick

from $\rho_{in} \sim 4 \times 10^{11} \text{ g/cm}^3$ to $0.5 \rho_0$

e^- 's, n -rich ions, n 's

n fraction increases as $\rho \uparrow$

nuclei $\rightarrow 0$ at crust/core boundary

nucleons can be superfluid if T low enough

magnetic effects
not important

~~Outer crust~~ ~~outer surface~~ ~~with gluons~~
protons & neutrons don't couple to each other
 $\frac{p}{\rho} \approx 20$
mp ratio in β equilibrium
many-body effects may be important
everything very degenerate
nucleon-nucleon interactions \Rightarrow n's superfluid
p's superconducting
contains (with inner core) $\approx \sim 99\%$ of mass

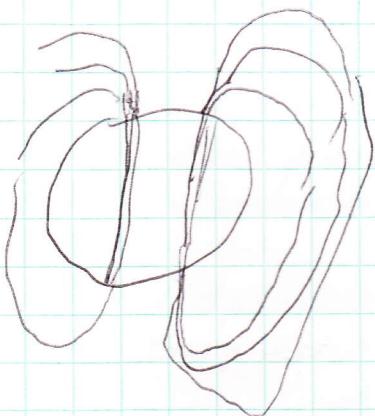
Inner core: $\rho \gtrsim 2\rho_c$ ρ up to $(10-15)\rho_c$?
 $R \gtrsim$ few km

composition/EoS uncertain; nobody knows what
nuclear matter at such high P_p does
hyperons? pions? kaons? quark matter?
may be probed in heavy-ion collisions
(e.g., RHIC @ Brookhaven)

Superfluidity: occurs for $T < T_c$ = critical temperature (uncertain)
does not affect EoS, M-R much
n's in inner crust should be superfluid
affects heat capacity and γ emission, and so
is relevant for cooling of NS.

Spin of NS is quantized in discrete vortices
superfluid p's are superconducting

B fields quantized in flux tubes
B fields fix crust/core rotation to be same,
but superfluid neutrons may have different spin.



Are born with $T \approx 10^9 K$ and radiate $\sim 10^{53} erg$

• few secs. $T \approx 10^8 K$ after ~day

• emission dominates for first 10^5 yr to $10^6 K$;

then X radiation from surface takes over

Observationally NS surface temperatures are $\approx 5 \times 10^5 K$
⇒ thermal emission in UV/soft-X-ray

γ 's produced with $E_\gamma \approx k_B T$ and scatter from N with
 $\sigma \approx 4.4 \times 10^{-45} cm^2 (E_\gamma / MeV)^2$,

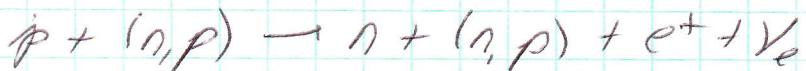
so their mean-free path is
$$l = \frac{1}{n\sigma} \approx 2500 km \left(\frac{R^3}{12 km} \right) \left(\frac{1.4 M_\odot}{m} \right) \left(\frac{10^9 K}{T} \right)^2$$

so @ $T \approx 10^9 K$ is opaque, but transparent for $T \lesssim 10^8 K$

Direct URCA process: $n \rightarrow p + e^- + \bar{\nu}_e \quad p + e^- \rightarrow n + \nu_e$.

is slow if p's are scarce; slowed also by Pauli blocking

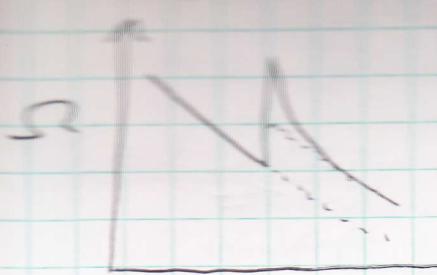
Modified URCA process:



can increase rate by providing more energy-momentum combinations.

Measurements of t (e.g., from P/\dot{P}) and T_{eff} (from X-ray)
suggest modified URCA is important.

Pulsar initially rapidly spins up and then
 slows down. This is explained by
 starquakes, cracks in the crystalline
 crust. Moment of inertia $I \downarrow$
 sending $\Delta\Omega/\Omega$. E.g., $\Delta\Omega/\Omega \sim 10^{-2}$
 once in Vela. More characteristically,
 $\Delta\Omega/\Omega \lesssim 10^{-6}$.



After glitch, pulsar spins down more rapidly.
 This is explained by the time it takes crust to
 spin superfluid-h component up. Then asymptotes to
 usual B-field spindown with slightly smaller I
 that results from quake.

Magnetars:

Typical NS B-fields are $T \sim 10^{12}$ G. But a few
 have $B \sim 10^{14}-10^{16}$ G \Rightarrow magnetars.

If B is large enough, then the Larmor radius $r_L = \frac{v_{MeC}}{eB}$
 of an e^- becomes smaller than the de Broglie wavelength,
 $\lambda = h/m_e v$. Thus, above a quantum-critical field

$$B_{QC} = \frac{m_e^2 c^3}{\pi e} = 4.4 \times 10^{13} \text{ G.}$$

QM effects become important

Example: Soft Gamma Repeaters (SGRs):

are x-ray sources that repeatedly emit γ -ray flashes;
 spectrum is softer than GRBs.

Have $P \sim 5-8$ s $\dot{P} \approx 7 \times 10^{-11}$

~~SGR 1806-20~~ → ~~SGR 1806-20~~

SGR has age $\sim 10^5$ yr and $P_{\text{orb}} \approx 5$

$$\text{using } \frac{P}{2\pi} = t \Rightarrow \dot{P} \approx 10^{-11} \text{ s}^{-1} \Rightarrow B \approx 10^{14} \text{ G}$$

Giant flares:

1806-20

SGR ~~1806-20~~'s most energetic burst was 27 Dec '04 with $E \approx 2 \times 10^{46}$ erg; (~~typical bursts are 10^{43} erg/sec~~) most released in \lesssim sec burst with 50 ms rise time rest released in softer pulsating tail

From PP, $B \approx 1.6 \times 10^{15}$ G.

$$E_{\text{mag}} \approx \frac{B^2}{8\pi} \frac{4}{3} \pi R^3 \approx 10^{48} \text{ erg},$$

so burst is only small fraction of B-field reservoir.

Explanation for pulsed emission is that starquake rearranges B-field produced γ -ray fireball confined to magnetosphere.

Anomalous X-ray Pulsars: (AXPs)

Isolated pulsars emitting pulsed x-ray with $L \approx 10^{35-36}$ $\frac{\text{erg}}{\text{sec}}$ and $P \approx 5-12$ sec.

L is too high for magnetic dipole; is assumed/guessed to be powered by dissipation of B field. Uncertain?