Magnetic Fields and Cosmic Rays

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The material on magnetic fields is assembled from several sources, but primarily from various places in Bruce Draine's book. The section on cosmic rays is taken primarily from Ch. 40 in Draine.

1 Flux freezing

The first thing to know is that magnetic fields are frozen into a plasma; i.e., the magnetic fields are preserved within and move with the plasma. To a very good approximation, the degree of ionization in almost any astrophysical system is sufficiently large, even in molecular clouds and HI regions, so that magnetic fields are flux frozen.

To see this, we start with Ampere's and Faraday's laws which are, respectively,

$$\begin{aligned} \nabla \times \mathbf{B} &= \frac{4\pi}{c}\mathbf{J} \\ \nabla \times \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \end{aligned} \tag{1}$$

and we also recall that the magnetic field is divergence-free, $\nabla \cdot \mathbf{B} = 0$. Note that we have neglected the displacement current, $(1/c)\partial \mathbf{E}/\partial t$, on the right-hand side of Ampere's law. You can convince yourself that if the fluid motions in question are all nonrelativistic, $v \ll c$, then the effect of this term are negligible. If the plasma is ionized, then there will be free electrons that can carry an electric current; i.e., there will be a finite conductivity σ. Free electrons are accelerated in the presence of $\mathbf{E}$ and $\mathbf{B}$ fields by the Lorentz force, or put another way, there is a current given by Ohm's law,

$$\mathbf{J} = \sigma \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right), \tag{2}$$

where $\mathbf{v}$ is the fluid velocity. We now solve this equation for $\mathbf{E}$ and substitute into Faraday's law to obtain,

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\frac{\mathbf{J}}{\sigma} - \frac{1}{c}\mathbf{v} \times \mathbf{B} \right). \tag{3}$$

We then eliminate $\mathbf{J}$ in favor of $\mathbf{B}$ with Ampere's law to obtain

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma}\nabla^2 \mathbf{B}, \tag{4}$$

where we have used $\nabla \times \nabla \times \mathbf{B} = -\nabla^2 \mathbf{B}$ (which follows from $\nabla \cdot \mathbf{B} = 0$).

Now consider a closed loop L that moves with the fluid, and let $\Phi(t)$ be the magnetic flux through that loop:

$$\Phi(t) = \int d\mathbf{S} \cdot \mathbf{B}, \tag{5}$$

where the integral is over a surface S bounded by the loop. The time derivative of the flux is

$$\begin{aligned}
\frac{d\Phi}{dt} &= \int d\mathbf{S} \cdot \frac{d\mathbf{B}}{dt} + \oint \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{L}) \\
&= \int d\mathbf{S} \cdot \frac{d\mathbf{B}}{dt} + \oint d\mathbf{L} \cdot (\mathbf{B} \times \mathbf{v}) \\
&= \int d\mathbf{S} \cdot \frac{d\mathbf{B}}{dt} + \int d\mathbf{S} \cdot \nabla \times (\mathbf{B} \times \mathbf{v}) \\
&= \int d\mathbf{S} \cdot \left[\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) \right] \\
&= \int d\mathbf{S} \cdot \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B}.
\end{aligned} \tag{6}$$

Thus, in the limit $\sigma \to \infty$, the flux through the loop is conserved; i.e., *magnetic-field lines move with the plasma.*

Still, the conductivity is finite. For $\mathbf{v} = 0$, we have $(\partial \mathbf{B}/\partial t) = (c^2/4\pi\sigma)\nabla^2 \mathbf{B}$, a diffusion equation with diffusion coefficient $c^2/4\pi\sigma$. Thus, the timescale for a magnetic field with a coherence length L to decay is roughly

$$\tau_{\rm decay} \simeq \frac{4\pi\sigma L^2}{c^2}. \tag{7}$$

The finite conductivity in a hydrogen plasma arises from electron-proton Coulomb scattering, and the conductivity is then

$$\sigma \simeq 0.59 \frac{(kT)^{3/2}}{e^2 m_e^{1/2} \ln \Lambda} = 4.6 \times 10^9 \, {\rm sec}^{-1} \left(\frac{T}{100\,{\rm K}} \right)^{3/2} \left(\frac{30}{\ln \lambda} \right), \tag{8}$$

where $\ln \Lambda$ is the Coulomb logarithm. We thus find

$$\tau_{\rm decay} \simeq 5 \times 10^8 \, {\rm yr} \left(\frac{T}{100\,{\rm K}} \right)^{3/2} \left(\frac{30}{\ln \lambda} \right) \left(\frac{L}{\rm AU} \right)^2. \tag{9}$$

The bottom line is that magnetic fields are very long-lived for ISM distance scales.

2 Evidence for ISM magnetic fields

2.1 Faraday rotation

A plasma has a frequency-dependent index of refraction. The speeds of electromagnetic waves that propagate through that plasma thus have a frequency-dependent velocity. If there is a source (e.g., a pulsar) that emits pulsed radiation, then the arrival times of those pulses will depend on the

frequency (and the time delay depends on ν^{-2}, where ν is the frequency). Measurement of these time delays can be used to infer the *dispersion measure*,

$$DM \equiv \int_0^L n_e \, dL, \tag{10}$$

where n_e is the electron density and the integral is carried out along the line of sight to the source. For example, a pulsar at a distance of 3 kpc might have a dispersion measure $DM \simeq 100\,\mathrm{cm}^{-3}$ pc, obtained from a pulse time delay ~ 0.4 sec.

If the plasma is magnetized, then there is an additional effect that acts on the polarization of the electromagnetic wave. Suppose there is a magnetic field in the plasma directed along the line of sight. Electrons will then spiral in one particular direction around this magnetic field, and so the indexes of refraction for right- and left-circularly polarized electromagnetic waves will differ. The propagation speeds for right- and left-circularly polarized waves will therefore differ slightly. Recalling that a linearly-polarized wave is a superposition of two circularly-polarized waves, the linearly-polarized wave will undergo something like a beat phenomenon that occurs when two waves of slightly different frequencies are superposed. What this results in is a rotation of the linear polarization of a linearly polarized EM wave by an angle, $\Psi = RM\,\lambda^2$, where λ is the wavelength and

$$RM = \frac{1}{2\pi}\frac{e^3}{m_e^2 c^4}\int_0^L n_e B_\parallel \, dL = 8.12\times 10^{-5}\frac{\int_0^L n_e B_\parallel dL}{\mathrm{cm}^{-3}\ \mu\mathrm{G\ pc}}\mathrm{rad\ cm}^{-2}. \tag{11}$$

Then, if the DM and RM are both measured, the electron-density–weighted mean line-of-sight magnetic field is

$$\langle B_\parallel \rangle = \frac{RM}{8.12\times 10^{-5}\,\mathrm{rad\ cm}^{-2}}\frac{\mathrm{cm}^{-3}\,\mathrm{pc}}{DM}\,\mu\mathrm{G}. \tag{12}$$

This can be measured along many different lines of sight, and also to pulsars at different distances along similar lines of sight, to get information about the three-dimensional magnetic field. Measurements indicate magnetic fields $B \sim 2-4\,\mu$G in the spiral arms and slightly smaller in the interarm regions, with a sign flip between arm and interarm.

2.2 Synchrotron radiation

Evidence for Galactic magnetic fields also comes from synchrotron radiation emitted by relativistic electrons moving in Galactic magnetic fields. There are two lines of reasoning: (1) The intensity of synchrotron radiation depends on both the magnetic-field strength and the energy density in relativistic electrons; as the magnetic-field strength is increased, the electron density is decreased and *vice versa*. The *total* energy density (magnetic field plus electron density) is minimized, and obtains a reasonable value, in "equipartition," the magnetic-field and electron energy densities are comparable.

(2) Synchtron radiation is polarized if the magnetic fields are coherent. If the magnetic field is homogeneous, then the polarization is $p = (\gamma+1)/(\gamma+7/3) \simeq 0.73$, where γ is the electron spectral index. Observations show $p = 0.1-0.2$, typically, a consequence of incoherence or Faraday depolarization, beam effects, etc. Still, the variation of the polarization across the sky can be used to infer a magnetic-field pattern in the Galaxy.

2.3 Zeeman shift

A magnetic field splits the electronic energy states of the same l into $2l+1$ energy levels split by energies $\sim \mu_B B \simeq 5.78 \times 10^{-15}\,(B/\mu\mathrm{G})$ eV, where $\mu_B = e\hbar/2m_e c$ is the Bohr magneton. Magnetic fields of strength $1-100\,\mu$G give rise to level splittings that are too small to be detected in sub-mm or shortward ($h\nu \gtrsim 10^{-4}$ eV). The splitting in the 21-cm line ($h\nu = 5.9\times10^{-6}$ eV) is split by $\sim 10^{-8}$ for a 10 μG field. This is smaller than the $v/c \sim 10^{-5}$ frequency shifts from velocity broadening in molecular clouds or the IGM and thus unobservable. However, Zeeman splitting gives rise to a difference in the frequencies of the two circular polarizations of the transition radiation, and these can be detected and have been used to measure magnetic-field strengths in HI regions in the ISM.

Diffuse clouds studied in 21-cm absorption this way have been found to have $B \simeq 6$ μG implying a magnetic pressure $B^2/8\pi k_B \simeq 10^4$ cm^{-3} K, several times larger than the gas pressure $nT \simeq 3000$ cm^{-3} K. Thus, magnetic fields may be dynamically important in HI regions. Zeeman splitting shows magnetic fields ~ 10 μG in the Orion A GMC. The HI regions in shell-like structures (e.g., North Celestial Loop, the North Polar Spur Loop, and the shell around the Eridanus superbubble) have ~ 10 μG fields whose pressure appears to be preventing the clouds from further compression by supernova shocks.

2.4 Polarization of starlight

Evidence that polarization of starlight was due to interstellar dust came from (a) the correlation between the magnitude of the polarization and the reddening, and (b) the coherence in the polarization between different stars in the same region of the sky. The polarization percentage peaks near the B band (5500 $\mathring{A}$) and follows the empirical Serkowski law,

$$p(\lambda) \simeq p_{\max} \exp\left[-K \ln^2(\lambda/\lambda_{\max})\right], \tag{13}$$

with $\lambda_{\max} \simeq 5500\,\mathring{A}$ and $K \simeq 1.15$. The peak polarization falls in the range $0 \leq p_{\max} \lesssim 0.03\,(A_V/\mathrm{mag})$.

Heuristically, the polarization arises if dust grains are elongated and somehow aligned. If so, then the absorption of light polarized along the long axis may differ from that along the short axis. The dust grains appear to be aligned by the interstellar magnetic field with their shortest axes parallel to the magnetic field; the mechanism is not well understood and is a subject of active current research. Since extinction increases toward the UV, while the polarization decreases, it suggests that the grains responsible for the polarization have radii $a \simeq 2(\lambda_{\max}/2\pi) \simeq 0.1\,\mu$m. In the UV, one moves to the geometric-optics limit, and both polarizations are absorbed similarly. Thus, the V band extinction must be due largely to $a \simeq 0.1$ μm grains, and these grains must be nonspherical and aligned with the magnetic field; and grains with $a \lesssim 0.05\,\mu$m, which dominate the exinction at $\lambda \lesssim 0.3\,\mu$m are either spherical (which seems unlikely) or minimally aligned.

3 Molecular clouds

In molecular clouds, magnetic fields can be measured further by Zeeman splitting in the OH Λ-doubling lines (1.665, 1.667, 1.720 GHz) or in the CN $1-0$ rotational transition (113 GHz). Fields strengths vary in molecular clouds from 0.1 to 3000 μG. There is a trend for larger B in regions of larger density (with n_H going from 10 to 10^7 cm^{-3}), but there is a large scatter in the B fields found at any fixed n_H. The trend implies, roughly speaking, Alfven speeds, $v_A = B/\sqrt{8\pi\rho} \sim (n_H/10^4\,\mathrm{cm}^{-3})^{0.15}$ km sec^{-1}. Comparing with the three-dimensional turbulent-velocity distribution σ_v implies $v_A/\sigma_v \simeq 0.75\,(n_H/10^4\,\mathrm{cm}^{-3})^{0.46}$. Thus, magnetic fields are dynamically important, and the turbulence in molecular clouds is more correctly MHD turbulence. Magnetic fields can also be mapped with polarization of background starlight or from polarization of 350 μm emission from dust.

Dynamically-important magnetic fields in molecular clouds can also be inferred from the Chandrasekhar-Fermi method. If the magnetic field were dynamically unimportant, then turbulet motions would disperse the magnetic-field directions. If the dispersion in the magnetic-field direction is found to be small, then the magnetic fields have enough "weight" that the are not pushed around by turbulence.

4 Alignment of Interstellar Dust Grains

Any theory for the alignment of dust grains with magnetic fields that is observed must account for (a) the alignment of the dust angular momentum with the magnetic field, and (b) the alignment of a principle axis of the body of the grain with the angular momentum. This problem received additional interest in the last decade or so with the CBI (Cosmic Background Imager; PI: Tony Readhead) discovery of excess emission in the ~ 30 GHz band. The explanation was dipole radiation from spinning dust, and if this was the explanation, then it implied that that emission might be polarized, a possible foreground for measurement of the CMB polarization.

4.0.1 Precession of the angular momentum about the magnetic field

The rotation periods are from milliseconds to less than a nanosecond (cf., the 30-GHz emission). The dust grain will generally have a magnetic moment parallel to the angular velocity for two reasons: (1) If the grain has an electric charge Q, which is usually distributed on the surface, the rotation gives rise to an magnetic-dipole moment $\mu = Qa^2\omega/3$. This gives rise to the precession of the angular momentum in the presence of a magnetic field, with precession frequency,

$$\Omega_L = \frac{5}{8\pi\rho} UB = 1.7 \times 10^{-8} \left(\frac{3\ \mathrm{g\ cm}^{-3}}{\rho}\right) \left(\frac{U}{\mathrm{Volt}}\right) \left(\frac{B}{5\,\mu\mathrm{G}}\right) \mathrm{sec}^{-1}. \tag{14}$$

This is a precession period $P = 2\pi/\Omega_L \simeq 10$ yr.

(2) But there is another, and usually stronger, effect, the *Barnett effect*, the spontaneous magnetization of a material due to the alignment of the spins of unpaired electrons with the angular velocity.

The energetics are such that the spin state parallel (rather than antiparallel) to the angular velocity is preferentially occupied. The induced magnetic moment is $\mu = -\chi V\hbar\omega/g\mu_B$, where $g \simeq 2$ is the gyromagnetic ratio, and $\chi \simeq 10^{-4}(20\ \mu\mathrm{K}/T_{gr})$ is the magnetic susceptibility of interstellar dust grains. The precession period is then

$$\begin{aligned}\tau_{\rm Barnett} &= \frac{2\pi}{\Omega_B} = \frac{2\pi I\omega}{\mu B} = \frac{4\pi g\mu_B\rho a^2}{5\hbar\chi B} \\ &\simeq 0.8\left(\frac{a}{0.1\,\mu\mathrm{m}}\right)\left(\frac{10^{-4}}{\chi}\right)\left(\frac{5\,\mu\mathrm{G}}{B}\right)\ \mathrm{yr},\end{aligned} \tag{15}$$

a very short period. Therefore, the properties (e.g., long-to-short axis ratio) of the dust grains should be averaged over the precession cone. Moreover, the timescales for change of the local magnetic field in the ISM are generally long compared with this timescale, and so the alignment of the grains will change with the magnetic field.

4.0.2 Alignment of the grain body with the angular momentum

The angular velocity is not generally required to be aligned with one of the principle moments of inertia of the grain. However, dissipation will tend to align it. Consider for example an oblate spheroid with moments of inertia $I_1 > I_2 = I_3$. The rotational kinetic energy is

$$E_{rot} = \frac{J^2}{2I} + \frac{J^2(I_1 - I_2)}{2I_1I_2}\sin^2\theta, \tag{16}$$

where θ is the angle between the angular momentum and the direction of the principle moment of inertia. This rotational energy is minimized when $\theta = 0$; i.e., when the angular momentum is aligned with the principle moment. If $\theta \neq 0$ initially, then the grain tumbles and dissipation (i.e., heating of the grain) can allow the energy to decrease, while conserving angular momentum, until the angular momentum is aligned. This would be true at zero temperature, but if the grain has a finite temperature, then detailed balance tells us that the alignment cannot be perfect. As long as the spin is suprathermal—i.e., $J^2/I \gg k_BT$—the grain alignment will be excellent.

4.0.3 Alignment of angular momentum with the magnetic field

For a long time, the favored hypothesis was the Davis-Greenstein mechanism. If the angular momentum is not aligned with the magnetic field, then as the dust grain rotates, it sees a time varying magnetic field. If the material is paramagnetic (as required for the Barnett mechanism to work), then the dust grain sees a rotating component of the magnetic field, and this gives rise to a rotating component to the induced magnetization of the grain. This time-varying magnetization dissipates (paramagnetic dissipation), converting that induced magnetic moment into heat, until the angular momentum is aligned with the magnetic field. The timescale for this dissipation is

$$\tau_{DG} = \frac{2\rho a^2}{5KB^2} = 1.5\times10^6\,a_{-5}^2\left(\frac{\rho}{3\,\mathrm{g\ cm^{-3}}}\right)\left[\frac{10^{-13}\,\mathrm{sec}}{K(\omega)}\right]\left(\frac{5\,\mu\mathrm{G}}{B}\right)^2\ \mathrm{yr}, \tag{17}$$

where $K(\omega) = \mathrm{Im}[\chi(\omega)/\omega] \simeq 10^{-13}\,(18\,\mathrm{K}/T_{gr})$ sec in terms of the complex susceptibility $\chi(\omega)$. Unfortunately, though, the mechanism doesn't work. If the angular momentum is purely thermal,

$J^2/2I \simeq k_B T$, then tandom collisions with gas atoms will tend to change the angular momentum on the timescale $\tau_M \simeq 1.45 \times 10^5 \left(\rho/3\,\mathrm{g\ cm^{-3}}\right) a_{-5}(30\,\mathrm{cm^{-3}}/n_H) T_2^{-1/2}$ yr required for the grain to run into its own mass of gas. If the Davis-Greenstein mechanism were the whole story, and if the rotation was thermal, then the alignment would be increasing with $\tau_M/\tau_{DG} \propto a^{-1}$. This would imply alignment of $a \lesssim 0.01\,\mu$m grains, but no alignment of $a \gtrsim 0.1\,\mu$m grains, opposite to what is seen.

Since Purcell (1979), it has been thought that suprathermal rotation, $J^2/2I \gg k_B T$, might solve the problem. He speculated that suprathermal rotation might occur if radiative processes at the surface of the grain, and/or ejection of molecular hydrogen, occurred anisotropically on the surface of the grain. Suppose, for example, we launched all our rockets from one launch pad, but that launch pad was skewed relative to the vertical. Then each time a rocket was launched, it would give the Earth the same angular-momentum kick. Each kick might be small, but launch enough rockets and the Earth would experience a significant torque. Unfortunately, though, further investigation suggests that the emission properties that would do this should change on timescales short compared with τ_{DG}. There is also a problem with thermal flipping, a process whereby the grain changes from one flip state, at constant angular momentum. In this case, if there was anisotropic emission fixed in body coordinates, then it would have a positive torque half the time and negative torque the other half.

The currently popular scenario is radiative torques. If the starlight background is anisotropic, then the inverse of some of the processes Purcell discussed could work, the only difference being that the torque in body-fixed coordinates would change sign upon a change in the flip state. This scenario is now empirically favored because it accounts for the observed decrease in polarized emission from deep within the molecular cloud (where the anistropic starlight has been presumably absorbed), and because it spins up the larger dust grains more effectively than the small ones.

5 Cosmic ray energy spectrum and composition

Most of the energy density in cosmic rays comes from ∼GeV protons, but there is a power-law energy distribution, $d\Phi/dE \propto E^{-\gamma}$ of cosmic-ray protons extending to $\gtrsim 10^{11}$ GeV. The spectral index is $\gamma \simeq -2.65$ from 10 GeV to $\sim 10^7$ GeV, and it then steepens (at the "knee") to $\gamma \simeq -3$. The cosmic rays at $E \lesssim 10^7$ GeV are probably accelerated in supernova remnants. The highest-energy cosmic rays are extragalactic in origin.

The vast majority of cosmic rays are protons, but there are all kinds of heavier elements, the next most abundant being helium. The elements Li, Be, and B are overabundant relative to what is produced in stars. The explanation is that heavier elements run into ISM protons and "spallate" into lighter nuclei, including these non-stellar nuclei. These spallation cross sections are known, and so we can infer from the abundance of these "secondary" nuclei relative to the primary ones that primary cosmic rays propagate through a ~ 6 g cm^{-2} of material before arriving. This is consistent, assuming they travel at the speed of light, with a propagation lifetime $\sim 10^7$ yr. Measurements of the relative abundances of various radioactively unstable isotopes obtains age estimates perhaps an order of magnitude larger, suggesting, perhaps, that the cosmic rays spend some fraction of the time outside the disk, where the ISM density is lower.

5.1 Acceleration of cosmic rays

The original suggestion of Fermi, now referred to as "second-order Fermi acceleration," now takes a back seat to "first-order Fermi" or equivalently, "diffusive shock" acceleration. We'll consider the latter. We *do* know empirically that charged particles are accelerated in astrophysical shocks; they are ubiqituous anywhere there are magnetic fields. They are known to have highly non-thermal power-law energy spectra. The basic idea, which we will go through, is correct, but detailed implementation in genuine astrophysical situations is extremely complicated and highly uncertain. One way to think of cosmic-ray acceleration is as follows: Magnetized astrophysical plasmas can be thought of as macroscopic objects that move with some velocity dispersion that defines a "temperature" $T \sim (1/2)Mv^2$, where M is the mass of a coherence region of plasma. Charged particles are microscopic things with energies E that bounce off of these macroscopic objects. According to equipartition, these scatterings should tend to equilibrate the two temperatures, and so the scatters tend to take the huge energy associated with macroscopic plasma motions and transfer it to the individual charged objects.

To be a bit more quantitative, let's suppose that there is a shock propagating through a magnetized plasma. Now suppose that there is a charged particle moving in the vicinity of the shock with a velocity greater than the shock velocity. The particle can scatter from magnetic-field fluctuations in both the post-shock and pre-shock regions; we are then to imagine a scenario where the particle bounces back and forth between the pre- and post-shock regions, passing through the shock each time it does so. In the frame of the shock, the pre- and post-shock fluids are both moving toward the shock, and the velocity difference between the two is $\Delta v = (1 - 1/r)v_s$, where $r = v_s/v_2$ is the shock compression ratio (and v_2 is the velocity of the post-shock fluid) parametrizes the strength of the shock. Each time the particle crosses the shock, it gets a momentum kick; the kinematics are exactly the same as the elastic bouncing of a ping-pong ball from a moving paddle. The momentum kick is $\Delta p = 2(\Delta v/3w)p$, where w is the original particle velocity. Averaging over all angles of incidence to the shock, this becomes $\langle \Delta p \rangle = 2(\Delta v/3w)p$. If the particle has a mean time t_{refl} before being reflected back, then the (inverse) acceleration time is

$$t_{acc}^{-1}(p) \equiv \langle (d/dt) \ln p \rangle = (1/p) \langle \Delta p \rangle t_{refl}^{-1} = (4/3)(\Delta v/w) t_{refl}^{-1}. \tag{18}$$

Each time the particle crosses the shock, it also has some probability to escape downstream from the shock. The mean time for this to happen is t_{esc}.

Let's now consider the momentum distribution $f(p)$ of particles near the shock. Conservation of particle number tells us that

$$\frac{\partial}{\partial p}(f\dot{p}) + \frac{\partial f}{\partial t} = -\frac{f}{t_{esc}}, \tag{19}$$

where the left-hand side is the total time derivative, and the right-hand side accounts for the escape of particles. In steady state, $\partial f/\partial t = 0$, and so

$$\frac{d}{dp}\left(f\frac{p}{t_{acc}}\right) = -\frac{f}{t_{esc}}. \tag{20}$$

If $dt_{acc}/dp = 0$, then

$$\frac{p}{f}\frac{df}{dp} = -\left(1 + \frac{t_{acc}}{t_{esc}}\right), \tag{21}$$

which has solution $f \propto p^{-\alpha}$ with $\alpha = 1 + t_{acc}/t_{esc}$.

The spectral index α is thus determined by the ratio t_{acc}/t_{esc}, and this can be determined as follows: The distribution $f(p)$ is constant across the shock. The flux of particles incident on the shock from far upstream will $(w/4)f(p)$, and the flux far downstream from the shock is $v_2 f$. The probability that a particle crossing the shock will escape downstream instead of being reflected back across the shock is therefore

$$\text{escape probability} = \frac{t_{esc}^{-1}}{t_{refl}^{-1}} = \frac{v_2 f}{wf/4} = \frac{4v_2}{w} = \frac{4v_s}{rw}. \tag{22}$$

We thus have

$$\frac{t_{acc}}{t_{esc}} = \frac{(3w/4\Delta v)t_{refl}}{(rw/4v_1)t_{refl} = \frac{3v_s}{r}\Delta v} = \frac{3}{r-1}. \tag{23}$$

The power-law index for cosmic rays is therefore $\alpha = (r+2)/(r-1)$, which evaluates to $\alpha = 2$ for a strong shock with $r = 4$, not too far from the $\alpha = -2.65$ observed. The discrepancy can be accounted for by noting that higher-energy cosmic rays are more likely to escape the Galaxy.

5.2 Injection problem

Fermi acceleration requires some "seed" population of low-energy ions to undergo acceleration, and the source of these particles remains a mystery. It has been speculated that the destruction of dust grains near the shock may provide the required ions. There may be some evidence for this in the data, as elements that are normally depleted into dust grains appear over-represented in cosmic rays (e.g., Mg, Fe, Si).

5.3 Upper limit to cosmic-ray energy

A charged particle moving in a magnetic field has a gyration radius $R = pc/eB = 3\times10^{-7}\,(pc/\text{GeV})(3\,\mu\text{G}/B)$ pc. Cosmic rays are confined to the acceleration region only by magnetic fields. Thus, if the size of the acceleration region is L, the maximum energy of a cosmic ray will be $E_{\max} = eBL$.

Supernova remnants produce strong shocks and have long been hypothesized to be the accelerators of Galactic cosmic rays. A supernova remnant with radius R has a compressed shell of thickness $L \simeq R/20$. Using this for L, and taking a characteristic radius at which a supernova shock becomes radiative, we infer a maximum energy for SNR-accelerated cosmic rays of $E_{\max} \simeq 10^7\,\text{GeV}(R/30\,\text{pc})(B/10\,\mu\text{G})$. The coincidence between this energy and that at which the knee in the cosmic-ray energy spectrum occurs is taken as evidence that SNRs have something to do with cosmic-ray acceleration. If this is the origin of cosmic rays, then order-of-magnitude estimates suggest that the cosmic-ray luminosity from SNRs is not too far below the total SNR energy output.

The same Fermi-acceleration processes that accelerate cosmic-ray protons and nuclei will also accelerate electrons. The strong synchrotron emission seen in SNRs is due to radiation from these cosmic-ray electrons, and fairly simple theoretical models show that the flux of synchrotron radiation is consistent with the luminosity one expects if Galactic cosmic rays are accelerated in SNRs.

Detailed calculations must take into account the fact that electrons are decelerated via emission of synchrotron radiation far more effectively than protons. These synchrotron losses occur not only in the SNR, but also in the propagation of cosmic-ray electrons once they leave the SNR and diffuse through the Galaxy. In the absence of synchrotron losses, electrons and protons would have similarly energy spectra and fluxes. Synchrotron losses steepen the electron energy spectrum and reduce the overall local flux. Thus, for example, at $E \simeq$TeV, the cosmic-ray electron intensity is $Ed\Phi/dE \simeq 1.2$ cm^{-2} sec^{-1} sr^{-1}, a factor 250 lower than the hadronic flux.

5.4 Cosmic-ray propagation

Once cosmic rays leave the SNR, they then propagate throughout the Galaxy. Since the gyration radii of $E \lesssim 10^7$ GeV cosmic rays are pretty small compared with the coherence lengths of ISM magnetic fields, they move to a first approximation along magnetic field lines. They may ocassionally move onto other magnetic field lines by scattering from MHD waves or from ISM nuclei. Even in the absence of such scattering, the magnetic-field lines may be tangled. In short, its a very complicated process, and very little reliable is known about the details. Theoretical models approximate all these complications by assuming that the cosmic rays simply diffuse through the Galaxy with some diffusion coefficient chosen to fit observational data. The flux of cosmic rays is locally isotropic (i.e., we don't see more coming from known SNRs), and so the diffusion approximation is probably justified.

Cosmic-ray protons with $E \lesssim 0.3$ GeV lose energy as they move through the ISM via ionization losses. The hydrogen column density required to stop a CR proton at these energies is (very roughly) $N_H \simeq 10^{26}\,(E/\mathrm{GeV})^2$ cm^{-2}. Thus, a 100-MeV proton can penetrate even a dense cloud, while a 1-MeV proton is excluded from even a diffuse HI cloud.

At energies $E \gtrsim 0.3$ GeV, cosmic-ray protons lose energy via pion production. These cosmic rays can scatter from ISM protons to produce pions through $CRp{+}ISMp \rightarrow p{+}p{+}\pi$. There are charged pions and neutral pions produced. The neutral pions decay immediately to $\pi^0 \rightarrow 2\gamma$. Gamma rays can also be produced by cosmic-ray electrons through bremmstrahlung (from scattering from ISM protons) or inverse-Compton scattering starlight. Gamma rays in the 50 MeV to 3 GeV range are produced primarily by cosmic rays with energies 1 to 10 GeV. The pion channel dominate the gamma-ray spectrum for $E > 150$ MeV and bremsstrahlung dominates at lower energies. The $E > 150$ MeV gamma-ray flux from an interstellar cloud is therefore proportional to the hydrogen column density of that cloud, and so observations of gamma rays can be used to determine this gas column density. The constant of proportionality can be calibrated by comparing with 21-cm emission, and it can then be used to calibrate the ratio between the CO $J = 1 - 0$ luminosity and gas mass.

5.5 ^{26}Al in the ISM

The ^{26}Al isotope undergoes a weak-process decay to an excited state of ^{26}Mg and a positron and neutrino with an energy release of 1.16 MeV with a half life of 7.4×10^5 Myr. That excited state of ^{26}Mg then decays quickly to the ground state via emission of a 1.81 MeV gamma ray.

This line has been mapped across the Galaxy by the INTEGRAL satellite, indicating a total mass $\sim 2.7\,M_\odot$ of ^{26}Al in the Galaxy. The origin of this isotope of aluminum is uncertain. Some of it may be produced by spallation of ^{28}Si or ^{40}Ar cosmic rays, but it is believed that the majority is produced in massive stars and/or core-collapse supernovae. If so, then the 1.81-MeV line maps the distribution of current star formation, since the decay timescale, $\sim 10^6$ yr, is short/comparable to the timescales for star formation.

5.6 Positrons in the ISM

Positrons are injected into the ISM by ^{26}Al decays at a rate $\sim 4 \times 10^{42}$ sec^{-1} and also by decays of some other unstable nuclei. High-energy positrons are also produced by cosmic-ray spallation. For each neutral pion produced by a cosmic-ray interaction with an ISM proton, there are two charged pions produced, and the positively charged ones decay to antimuons which then decay to positrons. They may be produced in the jets from compact objects, and there are also reasons to believe that energetic positrons may be injected into the ISM by pulsars.

Cosmic-ray positrons are slowed, like electrons, as they propagate through the ISM by synchrotron radiation, inverse Compton scattering, and Coulomb scattering. They can then annihilate directly with free electrons ($e^+ + e^- \rightarrow 2\gamma$), through electrons in hydrogen atoms ($e^+ + H \rightarrow H^+ + 2\gamma$), or formation of positronium (with free electrons) followed by decay of positronium.

Positronium is a hydrogen-like bound state of an electron and positron. The bound-state energies are lower than those of hydrogen by a factor of two because the reduced mass is lowered by 2. Positronium forms by radiative recombination (like recombination), $e^+ + e^- \rightarrow Ps + h\nu$, or by charge exchange, $e^+ + H \rightarrow Ps + H^+$, the latter requiring an activation energy of 6.8 eV.

The electron-positron spins can combine either into a triplet state (^{3}S$_1$, orthopositronium, with spin $S = 1$) or singlet state (^{3}S$_0$, parapositronium, with spin $S = 0$) with a population ratio 3:1. Orthopositronium decays to 3 photons with a lifetime $\tau = 1.4 \times 10^{-7}$ sec; these photons have a continuum energy spectrum extending up to 511 keV. Parapositronium decays to two 511-keV photons with lifetime $\tau = 1.25 \times 10^{-10}$ sec. The gamma-ray spectrum from positronium decay thus has a continuum from 0 to 511 keV with a 511-keV line with 1/3 the total intensity superposed. This spectrum has now been observed.

There has been considerable attention on positrons recently for two reasons: (1) PAMELA, an Italian satellite experiment, measured the spectrum of high-energy ($E \gtrsim$ 10s of GeV) cosmic-ray positron spectrum, and they find an excess over the expectations of standard cosmic-ray propagation models. Explanations for the discrepancy include (a) uncertainties in the propagation models; (b) previously unconsidered astrophysical sources (e.g., nearby pulsars); (c) dark-matter annihilation, this last explanation requiring several leaps of faith. (2) INTEGRAL measurements of the flux of positronium-decay gamma rays from the Galactic center are also larger than expected—there seems to be more positronium near the Galactic center than standard models for astrophysical positron production predict. Again, this is probably a shortfall of the models.