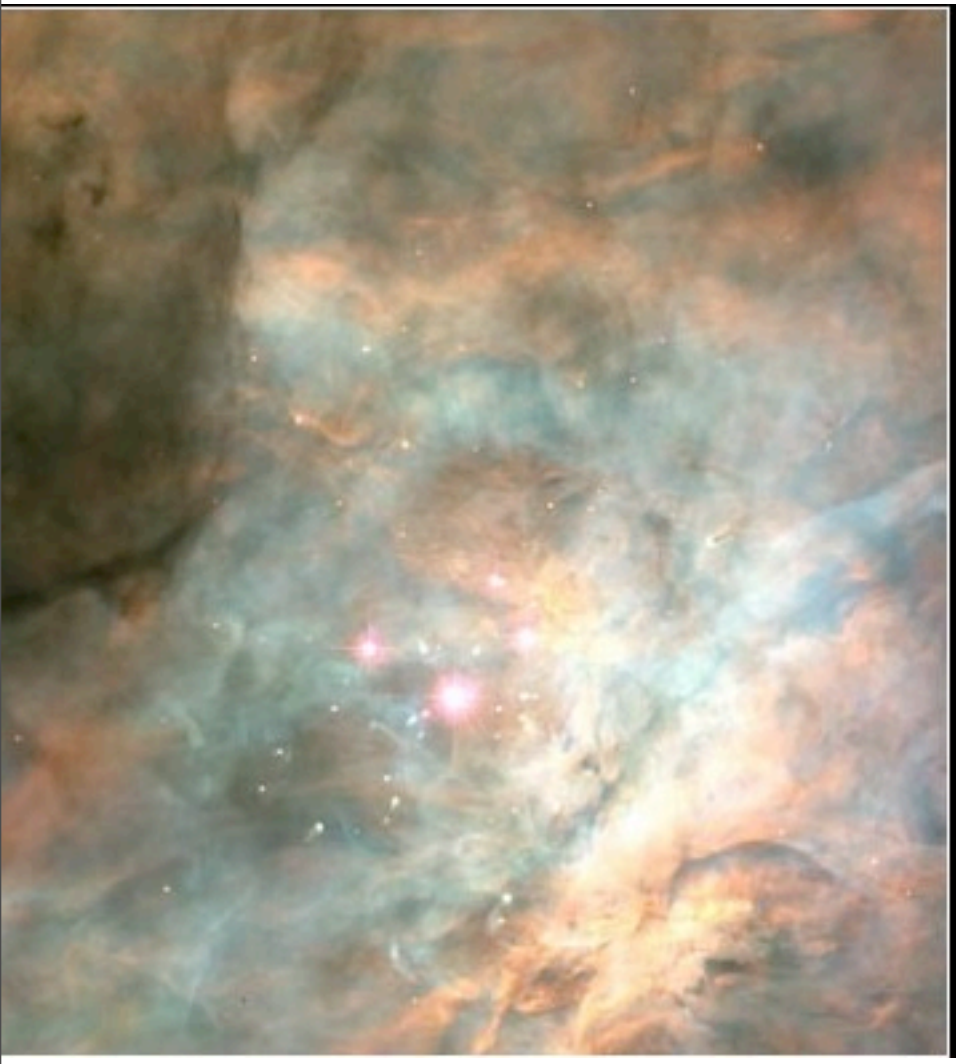
The Theory of Photoionized Regions

Trapezium cluster in Orion



HST Optical

HST IR

Table 7.1 *Stellar parameters of O and B stars*[a]

Spectral type	$T_{\rm eff}$(K)	$L(10^5 L_\odot)$	$\mathbb{N}_{\rm Lyc}(10^{49}$ photons s$^{-1})$	$\mathcal{R}_{\rm s}^b$ (pc)
O3	51 200	10.8	7.4	1.3
O4	48 700	7.6	5.0	1.2
O5	46 100	5.3	3.4	1.0
O6	43 600	3.7	2.2	0.88
O7	41 000	2.5	1.3	0.75
O8	38 500	1.7	0.74	0.62
O9	35 900	1.2	0.36	0.49
B0	33 300	0.76	0.14	0.36

[a] Stellar parameters for main sequence stars.

[b] Strömgren radius calculated for a density of 10^3 cm^{-3}.

HI in Photoionized Regions

Here we discuss photoionized regions (as opposed to collisionally ionized shocked regions).

In any photoionized region the column density of neutral hydrogen must be very low. Hydrogen, by far the most common component of the ISM, dominates the opacity above the Lyman limit (ν_0).

$$\kappa_\nu = \sigma_0 \left(\frac{\nu}{\nu_0} \right)^{-3.5} n_H$$

HI in Photoionized Regions

$$\kappa_\nu = \sigma_0 \left(\frac{\nu}{\nu_0} \right)^{-3.5} n_H$$

Where n_H is the volume density of atomic hydrogen and σ_0 is the absorption cross section at ν_0 (6.3×10^{-18} cm^2).

Putting in a typical value for n_H, one finds that an optical depth of τ=1 is achieved within ~0.1 pc at ν_0, and 0.6 pc at the threshold of ionization of helium.

Since HII regions are observed to have radii up to 100 pc, ~99% of the hydrogen in HII regions must be ionized.

Heating and Cooling

The local heating rate (dQ/dt) is controlled by the absorption of EUV photons by trace amounts of neutral hydrogen.

The local cooling rate is provided mostly by collisional excitation of forbidden emission lines in heavier atomic species.

$$\dot{Q} = \int_{\nu_0}^{\nu_{\max}} \frac{(\nu - \nu_0)}{\nu} I_\nu \kappa_\nu d\nu = \Lambda n n_e$$

Where the first term in the integral arises because the electrons produced by photoionization have to overcome the ionization potential of the H atom and Λ is the cooling function.

Heating and Cooling

The cooling rate increases rapidly with temperature (collisions become more likely). The heating rate decreases slowly with temperature because:

- the recombination rate deceases with temperature
- collisional ionizations deplete the population of neutral hydrogen, which lowers κ_ν

Because of this thermostatic effect, most HII regions are found to have temperatures around 10^4 K.

Local Radiation Field

The local radiation field is determined by radiative transfer through the nebula. For a simple plane-parallel nebula, this is given by:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

Where the j_ν source term refers to the recombination of hydrogen atoms from the continuum directly to the ground state, producing a photon above the Lyman limit.

Photoionization in an H-only Nebula

Let's consider a nebula composed only of hydrogen. In equilibrium, the rate of recombinations must match the rate of photoionizations. Neglecting the local radiation source term, we get:

$$\alpha(T_e) n_{H^+} n_e = \int_{\nu_0}^{\nu_{\max}} \kappa_\nu \frac{I_\nu}{h\nu} d\nu$$

$$\alpha(T_e) n_{H^+} n_e = n_{H^0} \int_{\nu_0}^{\nu_{\max}} \sigma_0 \left(\frac{\nu}{\nu_0}\right)^{-3.5} \frac{I_\nu}{h\nu} d\nu$$

Where $\alpha(T_e)$ is the effective recombination rate for hydrogen.

Photoionization in an H-only Nebula

Now let's simplify the integral to $S\langle\sigma\rangle$ where S is the number of source photons passing through through a unit volume locally and $\langle\sigma\rangle$ is the average photoionization cross section, weighted according to the shape of the ionizing source spectrum. S has units of $cm^{-2}s^{-1}$, and $\langle\sigma\rangle$ has units of cm^2.

Let's set χ = fractional ionization of hydrogen, so we set $n_e = n_{H+} = \chi n$ and $n_{H0} = (1-\chi)n$

$$\frac{\chi^2}{1-\chi} = \frac{\langle\sigma\rangle}{\alpha(T_e)}\frac{S}{n}$$

Photoionization in an H-only Nebula

$$\frac{\chi^2}{1-\chi} = \frac{\langle \sigma \rangle}{\alpha(T_e)} \frac{S}{n}$$

It turns out that $<\sigma>$ and $\alpha(T_e)$ only vary by a factor of a few, while S and n can take nearly any values, so the local ionizaton state of a plasma is primarily determined by the ratio S/n, which is called the *ionization parameter (q)*. Note that q has units of velocity.

Expanding H-only Nebula

Instead of being in equilibrium, imagine that the photon field S is incident on atomic hydrogen density n. In this case, the flux of photons will match the number of ***new*** ionizations they produce, so that the boundary of the ionized region will advance at velocity dx/dt given by:

$$\frac{S}{n} = \frac{dx}{dt}$$

Thus, q is the velocity of the ionization front that the radiation field will drive through the neutral medium.

Expanding H-only Nebula

The *dimensionless ionization parameter* is given by:
U = q/c, and is equal to the local ratio of the density of photons to the density of atoms.

Another dimensionless ionization parameter is Ξ, which is the ratio of the radiation pressure to the gas pressure. This is a useful parameter when considering the phase stability of the ISM.

$$\Xi = \left(\frac{P_{rad}}{P_{gas}}\right) = \left(\frac{\langle h\nu \rangle S/c}{2nkT_e}\right) = U\frac{\langle h\nu \rangle}{2kT_e}$$

Expanding H-only Nebula

If we define the "edge" of the region of ionized gas to be where $\chi = 1/2$, then defining $\langle\tau\rangle$ as the optical depth from the edge to another point within the ionized region gives us:

$$\langle\tau\rangle = \int_0^x \langle\sigma\rangle n(1-\chi)dx = \int_0^\lambda (1-\chi)d\lambda$$

Where we have introduced a dimensionless scale length $\lambda = \langle\sigma\rangle nx$. These equations provide a readily integrable approximate solution to the ionization balance in a plane-parallel slab of ionized hydrogen for any incident radiation field intensity.

Strömgren Radius

In a spherical nebula centered on the exciting source of radiation, if we assume that the hydrogen is fully ionized out to a radius R_s, outside of which it is not ionized, then we can equate the number of ionizing photons with the number of recombinations.

$$S_* = \frac{4\pi}{3}\alpha(T_e)n_{H^+}n_e R_s^3$$

$$R_s = \left[\frac{3S_*}{4\pi\alpha(T_e)n^2}\right]^{1/3}$$

Strömgren Radius

$$R_s = \left[\frac{3S_*}{4\pi\alpha(T_e)n^2} \right]^{1/3}$$

Note that the radius is inversely proportional to the density. However, for a given S_*, a larger HII region is harder to observe. The total flux is proportional to S_*, so a larger radius implies a lower surface brightness.

Final Stromgren Radius

Since the temperature inside the H II region is about 100 times that outside, the pressure inside is about 100 times that in the surrounding neutral gas. Thus, the H II region will expand until the pressures are equalized. So the ``final Stromgren Radius'' will be:

$$R_{final} = \left(\frac{3S_\star}{4\pi\alpha(T_e)n_{final}^2}\right)^{1/3}$$

where $2n_{final}T_{HII} = n_{HI}T_{HI}$ so that:

$$\frac{R_{final}}{R_{initial}} = \left(\frac{2T_{HII}}{T_{HI}}\right)^{2/3}$$

If $T_{HII} = 10,000deg., T_{HI} = 100deg.$ $\frac{R_{final}}{R_{initial}} = 34$

An H II typically expands at about 10 km/s so in practice it might take ~10^7 years to reach the final radius. An O star does not last this long and ends its life as a SN explosion inside an H II region.

Ionization vs. Radius

For a spherical nebula, we can determine the ionization balance from our equation for $\chi^2/(1-\chi)$ by integrating from radius r = 0, and allowing for the spherical divergence of the radiation field and its attenuation through the ionized volume.

Mean optical depth of a sphere of neutral H out to R_s

$$\frac{\chi^2}{1-\chi} = \frac{\langle\sigma\rangle}{\alpha(T_e)} \frac{S_* e^{-\tau(r)}}{4\pi r^2 n}$$

$$\frac{\chi^2}{1-\chi} = \frac{\tau_s}{3} e^{-\tau(r)} \left(\frac{r}{R_s}\right)^{-2}$$

Where $\tau_s = \langle\sigma\rangle n R_s$ and $\tau(r) = \int_0^r \langle\sigma\rangle n(1-\chi)dr$

Ionization vs. Radius

Because of the steep dependence of hydrogen opacity on frequency, the photons with energies just above the Lyman limit are absorbed first. The radiation field is therefore "hardened" during its passage through the nebula.

Although the density of photons is decreasing, the energy per ionization is increasing. In some cases the electron temperature increases with radius even though the ionization fraction is falling.

j_ν

We have, until now, neglected the j_ν term in the equation of transfer. However, we know that there is a local production of a diffuse ionizing radiation field resulting from recombinations of hydrogen directly to the ground state.

In normal HII regions, the optical depth of these photons is very high (Case B), so these diffuse photons will be re-absorbed within the nebula close to where they were produced.

In this case, the ionization state of the plasma is higher than previously calculated.

Ionization Fronts

Let us consider an HII region evolving around a newly-formed star. In this case only a fraction of the EUV photons emitted by the central source are used to maintain the ionization in the nebula and the remaining EUV photons push an *ionization front* through the neutral medium at a velocity $(dr/dt) = q$.

If we assume that the ionized part of the nebula is fully ionized and in equilibrium, then we get:

$$4\pi r^2 n \frac{dr}{dt} = S_* - \frac{4\pi}{3}\alpha(T_e) n^2 r^3$$

Ionization Fronts

$$4\pi r^2 n \frac{dr}{dt} = S_* - \frac{4\pi}{3}\alpha(T_e)n^2 r^3$$

Let ξ be a dimensionless distance variable and τ be a dimensionless time variable given by:

$$\xi = r / R_s$$

$$\tau = t / \tau_s$$

$$\tau_s = \frac{1}{\alpha(T_e)n}$$

Strömgren Timescale - recombination timescale for the ionized plasma ~$10^5/n$ yr

Ionization Fronts

With these definitions, the equation

$$4\pi r^2 n \frac{dr}{dt} = S_* - \frac{4\pi}{3}\alpha(T_e)n^2 r^3$$

…simplifies to:

$$\frac{d\xi}{d\tau} = \frac{(1-\xi^3)}{3\xi^2}$$

This has the solution:

$$\xi = [1 - e^{-\tau}]^{1/3}$$

Ionization Fronts

$$\frac{d\xi}{d\tau} = \frac{(1-\xi^3)}{3\xi^2} \qquad \xi = [1-e^{-\tau}]^{1/3}$$

We see that the advance of the ionization front is initially very rapid and varies as the inverse square of the radius, but eventually slows.

For an O star, when $\tau = 1$, the velocity of the ionization front can be approximated by:

$$v_{IF}(\tau = 1) \sim 57\left(\frac{S_{49}}{n}\right)^{1/3} \text{km/s, where } S_{49} = S_*/10^{49}\ \text{sec}^{-1}.$$

Expansion Stages

The processes of ionization and heating of the plasma result in a large jump in gas temperature (from ~10^2 K to ~10^4 K), which results in a jump in gas pressure (by a factor of ~100).

However, the expansion speed of the ionization front is much higher than the sound speed (~1 km/s) of the ionized or atomic gas. Therefore, neither the ionized nor the neutral gas can react dynamically to the increased pressure.

Expansion Stages

Stage 1:

In a fast ionization front, the ram pressure of the gas entering the front is matched by the sum of the gas pressure and ram pressure of the ionized gas leaving the front.

Thus, the final velocity of the gas relative to the ionization front is lower than the initial velocity, and the gas is compressed in the passage through the front. This is called an *R-type* front.

Expansion Stages

Stage 2:

As the HII region expands, the velocity of the ionization front falls until it approaches the sound speed in the ionized gas (but is still supersonic with respect to the atomic medium). This occurs after ~2 recombination times ($2\times10^5/n$ years).

Now the plasma can adjust to the steep pressure gradient and the density change across the ionization front is much greater. At a certain velocity the hot plasma will push a strong compression shock into the atomic gas ahead of it.

Expansion Stages

Stage 3:

As the expansion slows further, the ionized region detaches a strong shock which propagates into the neutral gas and we have a *D-type* front. The gas ahead of the ionization front is now denser than the ionized gas behind it.

The strength of the shock decays over time because the expansion velocity is decreasing. Cooling may produce an isothermal shock with the neutral gas compressed by a factor of ~100. The pressure in the post-shock gas is now matched to the pressure in the ionized region. The expansion velocity is subsonic with respect to the ionized plasma.

Expansion Stages

Stage 4:

Eventually the expansion velocity is subsonic relative to the ionized and neutral phases. Dynamical adjustments have allowed the pressure in the ionized plasma to fall towards the pressure in the neutral plasma so that the final Strömgren radius is very much greater than the initial radius.

However, the time required to reach this phase is generally longer than the stellar lifetime. Also, we have neglected the effects of energetic stellar winds, which dominate the dynamical effects of the ionization front at these late stages.

Jump Conditions

Mass flux equation:

$$\rho_0 v_0 = \rho_1 v_1 = m_H S$$

Mass flux controlled by the photon flux

Sound speed

$$c_s = \sqrt{\gamma P / \rho}$$

$$c_s = \sqrt{\gamma k T / m}$$

Momentum (pressure) equation:

$$\rho_0 \left(\frac{c_0^2}{\gamma} + v_0^2 \right) = \rho_1 \left(\frac{c_1^2}{\gamma} + v_1^2 \right) + \frac{h \langle \nu \rangle S}{c}$$

P_0 P_1 P_{rad}

Jump Conditions

Energy conservation equation:

$$\frac{\gamma}{\gamma-1}\frac{P_0}{\rho_0}+\frac{v_0^2}{2}=\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}+\frac{v_1^2}{2}-\frac{h\langle\nu-\nu_0\rangle S}{m_H}$$

$$c_1^2=c_0^2+\frac{\gamma-1}{2}v_0^2-\frac{\gamma-1}{2}v_1^2-(\gamma-1)\varepsilon$$

$c_0 \sim 1$ km/s

Mean kinetic energy per unit mass liberated in a photoionization

Jump Conditions

Ignoring the radiation pressure term we get:

$$\left(\frac{v_1}{v_0}\right)^2 - \left(1 + \frac{1}{\gamma M_0^2}\right)\left(\frac{v_1}{v_0}\right) + \frac{1}{\gamma M_1^2} = 0$$

$$M = v / c$$

Large M is the R-type solution, and small M is D-type:

$$v_R > \frac{2c_1}{\gamma^{1/2}} \qquad v_D < \frac{c_0^2}{2\gamma^{1/2} c_1}$$

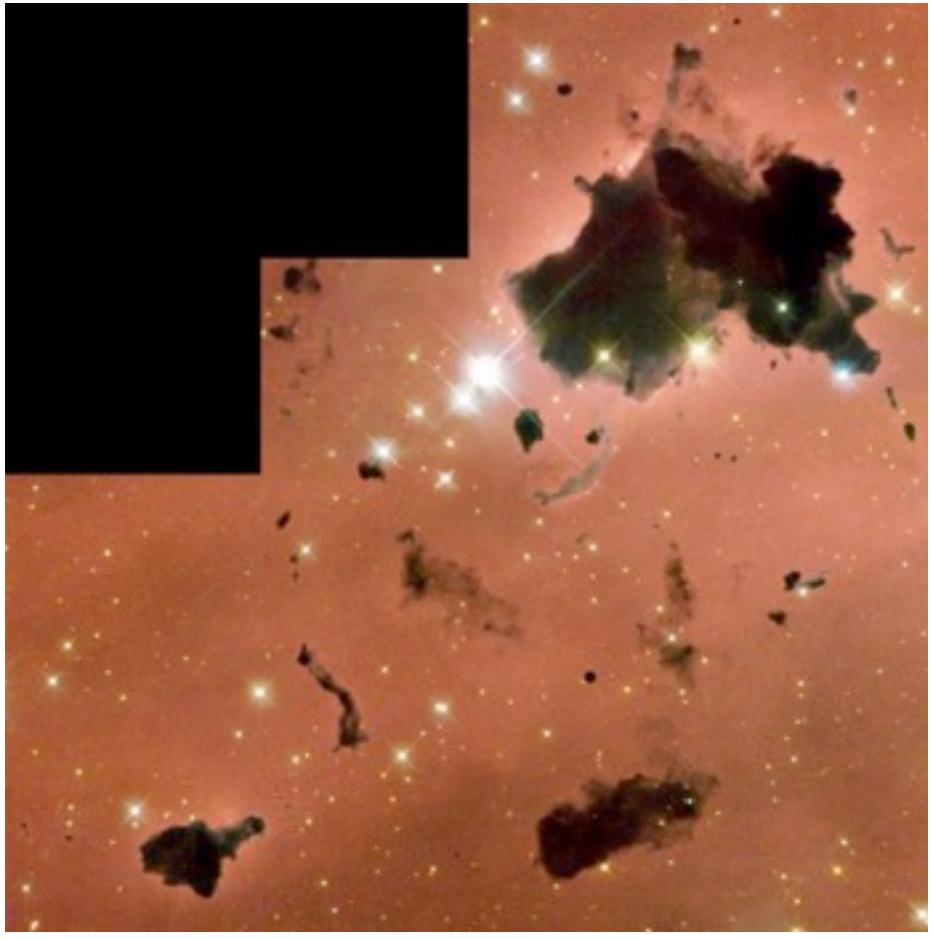
Photoionization of Neutral Condensations

Here we consider the photoionization of a dense neutral condensation within an HII region. Such condensations can be long lived for two reasons:

1. The layer of ionized gas streaming off the condensation shields the cloud, because much of the photoionizing radiation is used to maintain the ionization of the flow region
2. The recoil momentum of the ionized gas compresses the neutral gas and reduces the cross section of the cloud to the ionizing radiation field

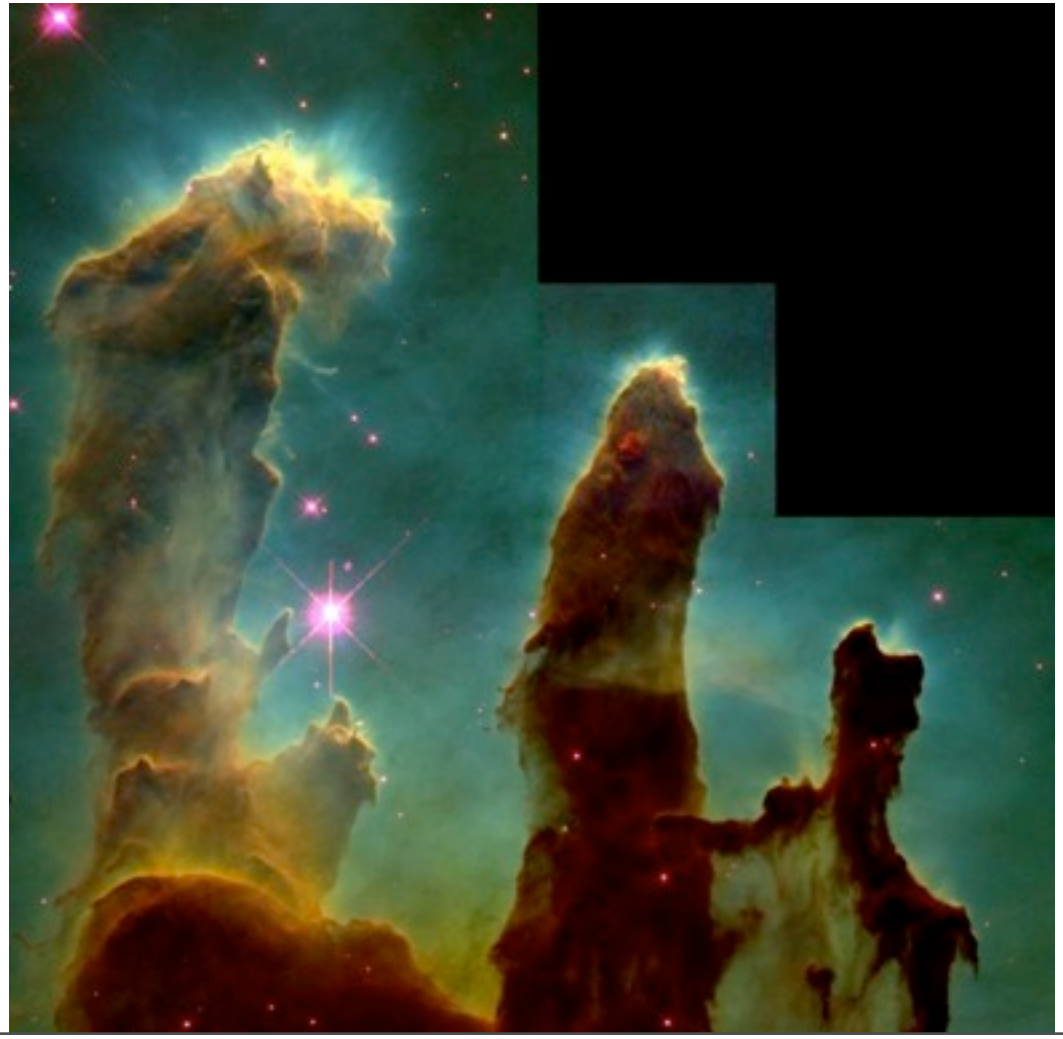
Photoionization of Neutral Condensations

Bok globules
in IC2944

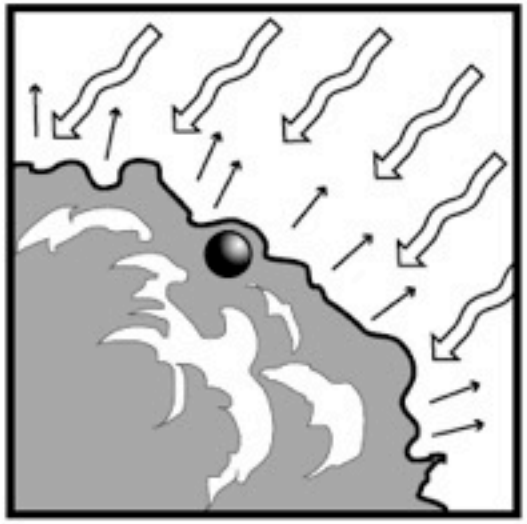


Photoionization of Neutral Condensations

Eagle Nebula



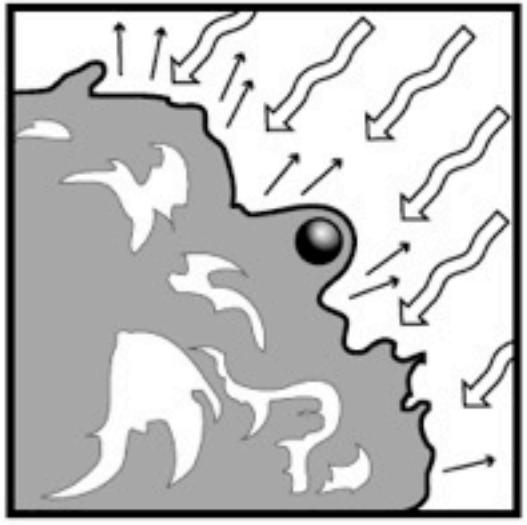
Photoevaporation



Stellar EGGs in M16

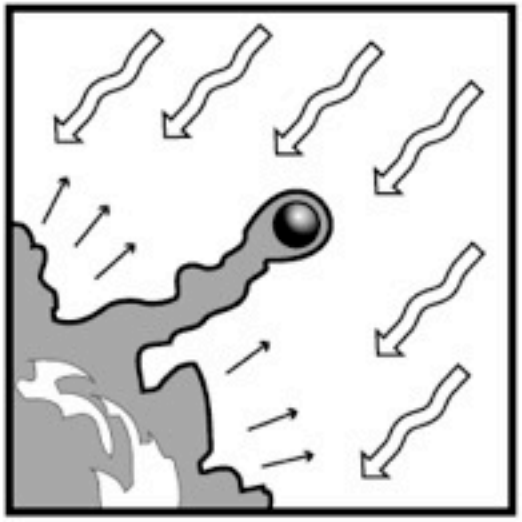
``Evaporating Gaseous Globule''

The surface of a molecular cloud is illuminated by intense ultraviolet radiation from nearby hot stars. The radiation evaporates material off of the surface of the cloud.

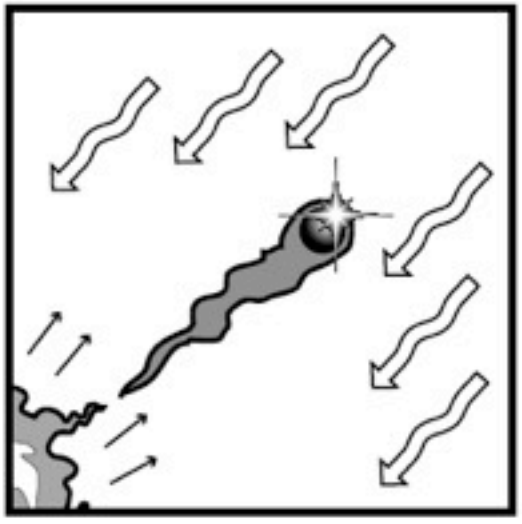


As the cloud is slowly eaten away by the ultraviolet radiation, a denser than average globule of gas begins to be uncovered

Photoevaporation



The EGG has now been largely uncovered. The shadow of the EGG protects a column of gas behind it, giving it a finger-like appearance.



Eventually the EGG may become totally separated from teh molecular cloud in which it formed. As the EGG itself slowly evaporates, the star within is uncovered and may appear sitting on the front surface of the EGG.

HII region with H and He

By including He in our HII region, we are adding in extra sources of opacity at energies high enough to singly or doubly ionize helium.

$$d\tau_\nu = -\kappa_\nu dx$$

$$\kappa_\nu = n_{H^0}\sigma_H(\nu) + n_{He^0}\sigma_{He}(\nu) + n_{He^+}\sigma_{He^+}(\nu)$$

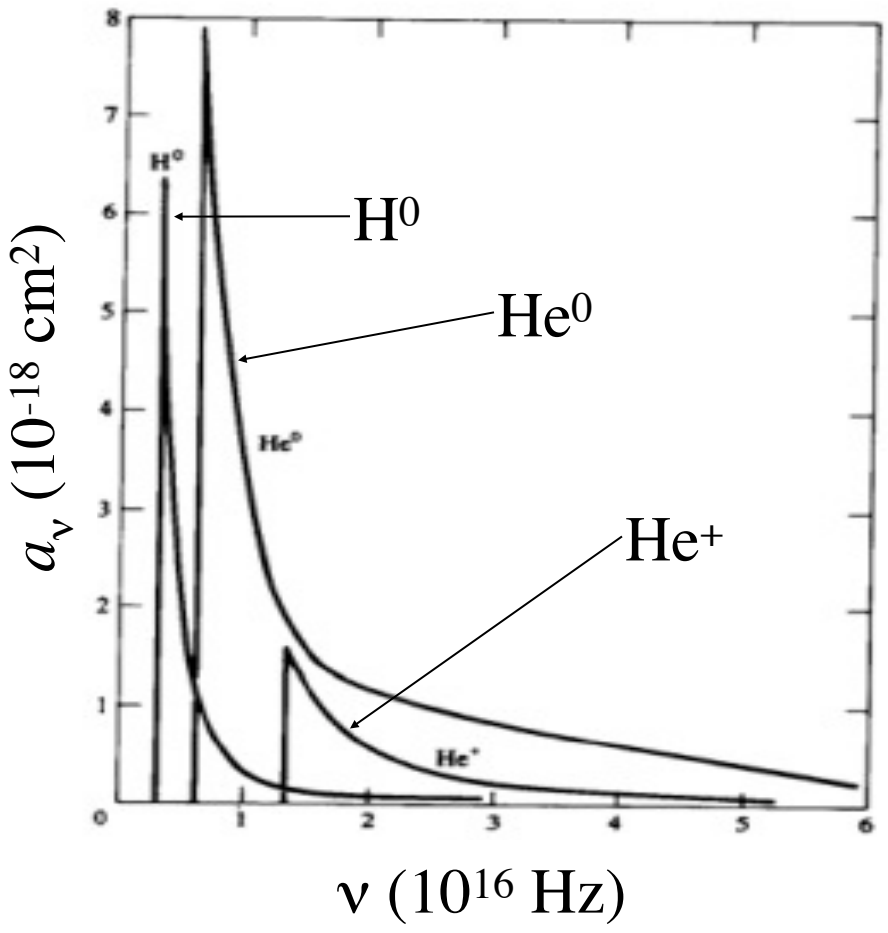
A-band: $1.0\nu_0 \leq \nu \leq 1.0\nu_0$: H^0-ionizing

B-band: $1.8\nu_0 \leq \nu \leq 4.0\nu_0$: He^0-ionizing

C-band: $4.0\nu_0 \leq \nu$: He^+-ionizing ← negligible

HII region with H and He

Figure 4.3 Photoionization absorption cross section of H^0, He^0, and He^+.



HII region with H and He

In addition to increasing the opacity, radiation from He recombinations and electronic transitions are also important.

Recombinations directly to the 1^1S ground state of HeI produce ionizing photons, and can not be treated with an on-the-spot approximation.

Transitions from the n=2-1 states produces an A-band photon (which occurs after every recombination).

He Strömgren Sphere

Here we calculate the radius of the region HeII, which is controlled by the radiative transfer of B-band photons. The helium ionization balance is given by:

$$S_* = S_A + S_B, Z_{He} = n_{He}/n_H$$

$$\frac{(\chi_H + Z_{He}\chi_{He})\chi_{He}}{1-\chi_{He}} = \frac{\langle \sigma_{He} \rangle}{\alpha_{He}(T_e)} \frac{S_B e^{-\tau_B(r)}}{4\pi r^2 n_H}$$

$$\tau_B(r) = \int_0^r n[\langle \sigma_H \rangle (1-\chi_H) + Z_{He}\langle \sigma_{He} \rangle (1-\chi_{He})]dr$$

Frequency-weighted mean photionization cross section to B-band photons for H and He

He Strömgren Sphere

The hydrogen ionization balance is given by:

$$\frac{(\chi_H + Z_{He}\chi_{He})\chi_H}{1-\chi_H} = \frac{\langle\sigma\rangle}{\alpha_H(T_e)}\frac{S_* e^{-\tau_A(r)}}{4\pi r^2 n_H}$$

$$\tau_A(r) = \int_0^r n\langle\sigma\rangle(1-\chi_H)dr$$

Frequency-weighted mean photionization cross section of H to A-band photons

He Strömgren Sphere

From the ratio of the ionization balance equations and assuming that $\chi \sim 1$,

$$\frac{S_B}{S_*} = \frac{S_B}{S_{A+B}} < \frac{\alpha_{He}(T_e)\langle\sigma\rangle}{\alpha_H(T_e)\langle\sigma_{He}\rangle}$$

This ratio is strongly dependent on the effective temperature of the ionizing source. The helium Strömgren sphere grows to fill the hydrogen Strömgren sphere as the temperature is increased. They are the same for $T_{eff} \sim$ 40,000 K.

The helium Strömgren sphere is never larger than the hydrogen Strömgren sphere.

Ionization with Heavy Metals

Heavy metals are significant coolants of HII regions, but they do not have much effect on the opacity. The important coolants of the ionization zones are:

HI, HeI	CII, NI, NeI, SII
HII, HeI	CII, (CIII), NII, OII, NeII, SII, (SIII)
HII, HeII	CIII, (CIV), NIII, OIII, NeIII, SIII, (SIV, SV)
HII, HeIII	CIV, NIV, OIV, NeIII, SV and higher

Where non-dominant ionization stages are indicated in parentheses

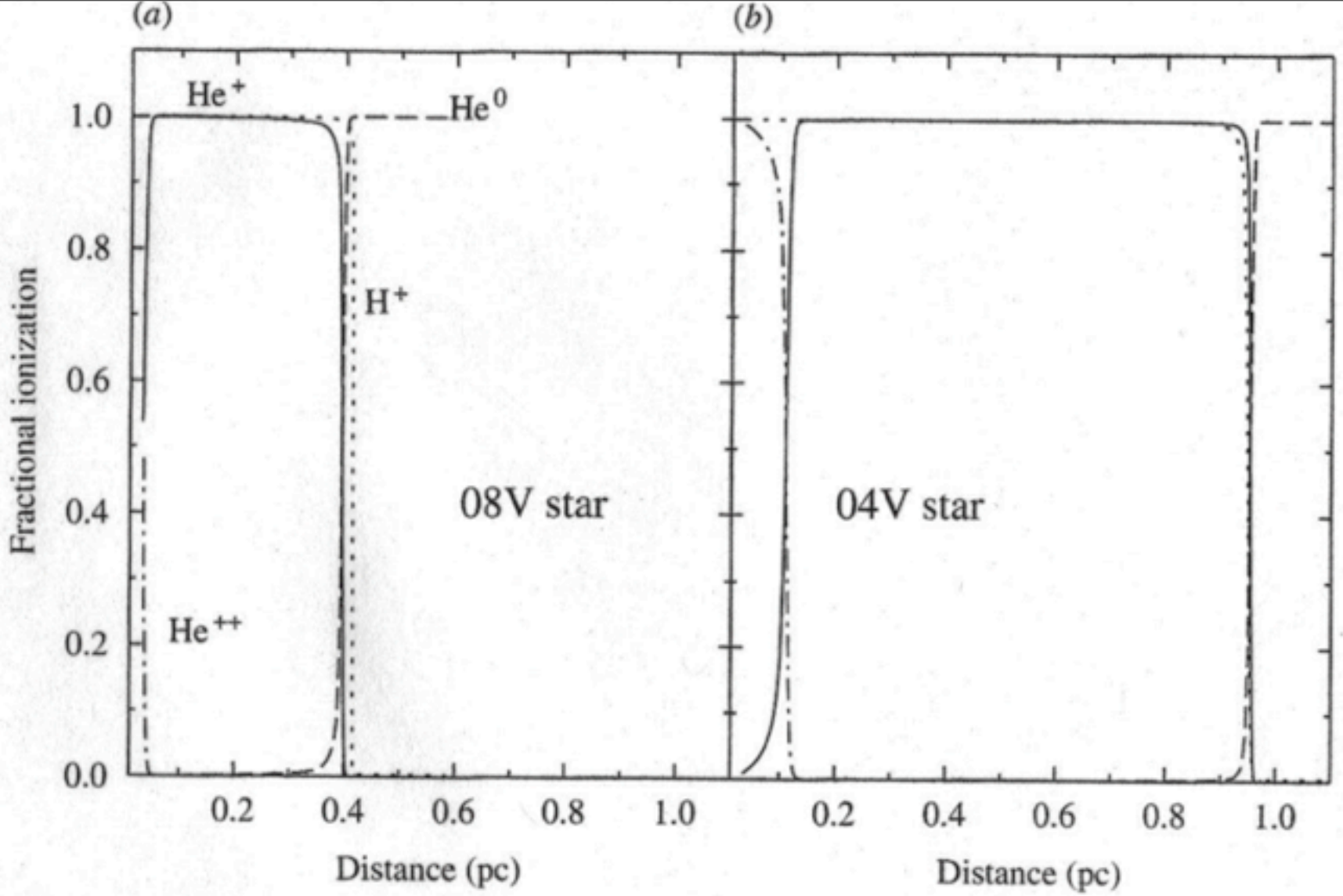


Figure 7.3 The calculated ionization structure of a nebula containing hydrogen and helium. For an O4 star (*a*), the H^+ and He^+ zones are essentially coincident. For an O8 star (*b*) and stars cooler than that, the He^+ zone has shrunk to less than the H^+ zone. Near the star, a small He^{++} zone is evident.

Ionization with Heavy Metals

The ionization structure is modified by charge-exchange reactions:

$$A^{(i+1)+} + H^0 \Leftrightarrow A^{i+} + H^+ + \Delta E$$

$$A^{(i+1)+} + He^0 \Leftrightarrow A^{i+} + He^+ + \Delta E$$

The reactions lock the ionizatoin ratio OII/OI so that it varies with the HII/HI ratio. The reactions also lower the OIII/OII ratio in the intermediate ionization zones.

Nebular Equilibrium Temperature

In an HII region, heating is determined by the local rate of photoionizations of H and He and the mean energy of the liberated photoelectrons. This is a function of the intensity and shape of the local ionizing radiation field: $\varepsilon(U, T_{\mathrm{eff}}, \tau)$

At low densities the cooling function is dependent on the local electron temperature, ionization state and heavy metal abundance: $\Lambda(T_e, Z)$

Nebular Equilibrium Temperature

Let's assume that we have local photoionization equilibrium, so that the photoionization rate is equal to the recombination rate. Let's ignore the recombination heating term. In this case, we have:

$$\dot{Q} = \varepsilon(U, T_{eff}, \tau) n_e [\alpha_H(T_e) n_H + \alpha_{He}(T_e) n_{He} + \alpha_{He^+}(T_e) n_{He^{++}}]$$

$$\dot{Q} = n n_e \Lambda(T_e, Z)$$

Nebular Equilibrium Temperature

For plasmas with solar or below heavy metal content, we can approximate that:

$n \sim nH + nHe = [1+Z(\mathrm{He})]\mathrm{nH}$

So, in the zone where only hydrogen is ionized, we get

$$\varepsilon(U,T_{eff},\tau)\alpha_H(T_e) = [1 = Z(He)]\Lambda(T_e,Z)$$

In the zone where hydrogen is ionized and helium is singly ionized:

$$\varepsilon(U,T_{eff},\tau)[\alpha_H(T_e) + Z(He)\alpha_{He}(T_e) = [1 = Z(He)]\Lambda(T_e,Z)$$

Nebular Equilibrium Temperature

For $T_e \sim 100$ K, cooling is dominated by ground-term fine structure splitting of the heavy metals in the mid and far-IR.

For $T_e \sim$ few$\times$1000 K, cooling is dominated by optical forbidden lines.

For $T_e \sim 15{,}000$ K, cooling is dominated by UV resonance lines.

Nebular Equilibrium Temperature

For nebulae with roughly solar metallicity, temperatures are ~10^4 K and the spectrum is dominated by optical forbidden lines.

For nebulae with low metallicity, temperatures are much higher and the spectrum is dominated by UV resonance and intercombination lines. The spectrum will be dominated by hydrogen and helium recombination lines.

For metal rich nebulae, $3.8 > \log T_e > 2.8$, which is too cool to excite the optical forbidden lines. The spectrum is dominated hydrogen and helium recombination lines.