Interstellar Medium (Ay126), Spring 2011

Final Exam

Due: 5pm, 18 March 2011 (hand in to Marc or to Shirley Hampton)

- The exam must be completed in a single sitting of no more than 4 hours.
- You may not communicate with anybody else about this exam (except perhaps with Wal or Marc if you have questions).
- You should answer 4 of the 6 short questions (each worth 10 points) and 3 of the 4 long problems (each worth 20 points) for a total of 100 possible points.
- A non-programmed calculator is permitted.
- Computers and programmable calculators are not permitted.
- You may use (and will almost certainly need) the textbook by Tielens as well as class notes, homework sets, and solutions for the class. Please do not refer to anything else.

Short Problems (do any 4 of these 6), 10 points each.

1. A pulsar is observed at 1610 and 1660 MHz. The plane of polarization at these two frequencies differs by 57.5°.

   (a) What is the *minimum* possible magnitude of the rotation measure $|RM|$ toward this source? Why is it a minimum? What would be the next-largest possible value of $|RM|$?

   (b) If the source has dispersion measure $DM = 200$ cm^{-3} pc, and using the minimum $|RM|$ from (a), what is the electron-density–weighted component of the magnetic field along the line of sight?

2. Sketch the frequency spectrum of the radiation field inside a photodissociation region, labeling the key contributors to the radiation field.

3. Show that neglect of the displacement current in Ampere's law is justified in the derivation of the ideal-MHD equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{B} + \frac{c^2}{4\pi\sigma}\nabla^2\mathbf{B}. \tag{1}$$

4. Suppose a cosmic-ray proton with energy E runs into an ISM proton at rest. How big does the energy E need to be to produce a neutral pion? What is the energy of the pion in laboratory frame produced at this threshold proton energy? What are the maximum and minimum photon energies that result from decay of this pion?

5. Outline how absorption lines in the spectra of stars are used to determine the density and temperature of interstellar gas. Why are O or B stars normally used for this purpose?

6. Estimate the length of time it takes for the recombination rate to balance the photoionization rate after the photons from a typical ionizing star are switched on.

Long Problems (do any 3 of these 5), 20 points each.

1. Both H_2 and HD have similar internuclear separation $r_0 \simeq 0.741$ $\AA$. Assume that the molecules can be approximated as rigid rotators.

   (a) Calculate $[E(v=0,J) - E(v=0,J=0)]/k$ for H_2 for $J=1$, $J=2$, and $J=3$.

   (b) Calculate $[E(v=0,J) - E(v=0,J=0)]/k$ for HD for $J=1$, $J=2$, and $J=3$.

   (c) Because H_2 has no electric dipole moment, $\Delta J = \pm 1$ transitions are forbidden, and instead the only radiative transitions are electric-quadrupole transitions with $\Delta J = 0, \pm 2$. Calculate the wavelength of the $J = 2 \rightarrow 0$ and $J = 3 \rightarrow 1$ transitions of H_2.

   (d) Because HD has a (small) electric dipole moment, it has (weak) electric-dipole transitions. What is the longest-wavelength spontaneous decay for HD in the $v = 0$ vibrational level?

2. Most interstellar CO is $^{12}C^{16}O$. The $J = 1 \rightarrow 0$ transition is at $\nu = 115.27$ GHz, or $\lambda = 0.261$ cm, and the $v = 1 \rightarrow 0$ transition is at $\lambda = 4.61$ μm.

   (a) Estimate the frequencies of the $J = 1 \rightarrow 0$ transitions in $^{13}C^{16}O$ and $^{12}C^{17}O$.

   (b) Estimate the frequencies of the $v = 1 \rightarrow 0$ transitions in $^{13}C^{16}O$ and $^{12}C^{18}O$.

   (c) Suppose that the $^{13}C^{16}O$ $J = 1 \rightarrow 0$ line were mistaken for the $^{12}C^{16}O$ $J = 1 \rightarrow 0$ line. What would be the inferred radial velocity of the emitting gas?

3. The total mass of neutral gas in the Galaxy is $\sim 4 \times 10^9$ $M_\odot$. Assume that it is uniformly distributed in a disk of radius $R_{disk} = 15$ kpc and thickness $H = 200$ pc, and that it is a mixture of H and He with He/H=0.1 (by number). Assume ionized hydrogen to be negligible in this problem.

   (a) What is the average number density of hydrogen nuclei within the disk?

   (b) If 0.7% of the interstellar mass is in the form of dust in spherical particles of radius $a = 1000$ $\AA = 0.1\,\mu$m and density 2 g cm^{-3}, what is the mean number density of dust grains in interstellar space?

   (c) Let Q_{ext} be the ratio of the visual (V band, $\lambda = 0.55\,\mu$m) extinction cross section to the geometric cross section πa^2. Suppose that $Q_{ext} \simeq 1$. What would be the visual extinction A_V (in magnitudes!) between the Sun and the Galactic Center (assumed to be 8.5 kpc away).

   (d) Now assume that 30% of the gas and dust mass is in spherical molecular clouds of radius 15 pc and mean density $n(H_2) = 100$ cm^{-3}. What would be the mass of one such cloud? How many such molecular clouds would there be in the Galaxy?

(e) With 30% of the gas and dust mass in molecular clouds as in (d), what is the expectation value for the visual extinction A_V to the Galactic Center?

(f) With 30% of the material in molecular clouds as in (d), what is the expectation value for the number of molecular clouds that will be intersected by the line of sight to the Galactic center? What is the probability that zero molecular clouds will be intersected?

(g) If the line of sight to the Galactic center happens not to intersect any molecular clouds, and if the atomic hydrogen and associated dust are distributed uniformly throughout the disk volume, what will be the visual extinction to the Galactic center?

4. A shock is moving at $10^4\,\mathrm{km\,s^{-1}}$ into neutral atomic hydrogen with $T_0 = 100\,\mathrm{K}$.

   (a) What are the sound speed in the unshocked gas and the Mach number of the shock?

   (b) Ignoring energy lost to ionization, what will be the temperature of the post-shock gas?

   (c) You are given the standard Rankine-Hugoniot jump conditions for a normal shock, where a 1 subscript labels a quantity on the unshocked side of the shock front and a 2 subscript labels a quantity on the post-shock side. Let the velocity c_* be defined by

   $$\left(\frac{\gamma+1}{\gamma-1}\right)c_*^2 = h + \frac{1}{2}u^2$$

   (note this is a quantity which is conserved across the shock).

   (d) Derive the Prandtl-Meyer relation: $u_1 u_2 = c_*^2$.

   (e) Prove that the sound speeds c_1 and c_2 before and after the shock obey $c_1 < c_* < c_2$.

   (f) Deduce from the Prandtl-Meyer relation that in a compressive shock, the upstream flow is supersonic and the downstream flow is subsonic.

5. (a) Describe in physical terms the stages in the evolution of a supernova remnant.

   (b) Estimate the typical lengths of time spent in each phase.

   (c) Explain qualitatively how a general interstellar magnetic effects the expanding shell.