Interstellar Medium (Ay126), Spring 2011

Problem Set 2

Due: Wednesday, 19 January 2011

1. **Cooling of the phases of the ISM**

   (a) Adopting the characteristics of the different phases in Tielens Table 1.1, estimate the cooling rate per atom in the HIM, WNM, and CNM from Figure 2.10. Typical electron (ionization) fractions to use are given in section 3.11.

   (b) Adopting the total masses of gas in these different phases given in Table 1.1 (for the total mass of the HIM, not given in column 5, by taking the total mass of the two WNM phases, and scaling by the ratio of the local surface densities given in the last column), estimate the total luminosities and the uncertainties in your estimates, for the HIM, WNM and CNM. You should get something like 3×10^{40}, 10^{41}, and 3×10^{41} erg/sec, respectively.

   (c) Estimate the total thermal energy content in each of the HIM, WNM and CNM phases.

   (d) Using your results from parts (b) and (c), estimate the cooling times of these phases (i.e. how long it would take them to cool if their sources of energy input were turned off). Use this to show that the ISM must be in a dynamical equilibrium.

2. **Heating of cool phases of the ISM**. In these calculations, answers are needed only to a factor of two or so. Dont bother doing detailed fits and numerical integrations on the computer. Do integrate-by-eye estimates.

   (a) Estimate the heating rate per H-atom due to CI ionization in an HI region due to the the average interstellar radiation field for a neutral carbon fraction f(CI). Adopt a mean CI photo-ionization cross-section of 10^{-17} cm^2 and a gas-phase carbon abundance of 10^{-4}, and estimate the mean CI-ionizing (> 11.26 eV or $\lambda < 1100\,\AA$) photon intensity and mean photoionizing photon energy from Figure 1.9 of Tielens. Compare your result to equation 3.8 of Tielens.

   (b) Estimate the photo-electric heating rate per hydrogen atom due to the ionization of neutral PAHs in the average interstellar radiation field. Adopt an ionization potential of 6 eV, a mean photo-ionization cross-section of 10^{-17} cm^2 per carbon atom, a fraction of the carbon locked up in PAHs of 0.05, and an elemental carbon abundance of 3.5×10^{-4}, and estimate the mean ionizing (> 6 eV) photon intensity and mean photoionizing photon energy from Figure 1.9 of Tielens. Compare your result to equation (3.17) of Tielens (recall that G_0 is the ratio of the mean UV interstellar flux to the value adopted by Habing, defined on page 13 of Tielens. You may insert a current estimate of $G_0 = 1.7$, consistent with Fig 1.9).

(c) Estimate the cosmic-ray heating rate. Adopt the interstellar proton cosmic-ray flux after correction for Solar-wind modulation from Fig 1.11 of Tielens. The rate of ionization energy loss cosmic rays protons of speed v, kinetic energy $m_p v^2/2$ experience as their electric fields move through hydrogen of density n is (in Gaussian units),

$$\frac{dE}{dt} = \frac{4\pi e^4 n}{m_p v} \times \ln X. \tag{1}$$

The logarithmic factor, of order a few, depends on the ionization potential and cosmic-ray energy (see e.g. Leighton *Principles of Modern Physics* for a derivation). The equation is approximately valid both for nonrelativistic and relativistic ($v \to c$) cosmic rays, and you should integrate (roughly) over the cosmic-ray spectrum. Compare your result to equation (3.31) of Tielens.

3. **Sample cooling detail of the cool phases of the ISM**. The rate coefficient for collisional de-excitation of the ground-state $^2P_{3/2} \to^2 2P_{1/2}$ fine-structure transition[1] of CII by collisions with neutral hydrogen atoms is 6×10^{-10} cm^3 sec^{-1}, nearly independent of temperature. The Einstein A coefficient for spontaneous radiative de- excitation is $A_{ul} = 2.3\times 10^{-6}$ sec^{-1}, and the photon emitted in the transition has wavelength 157.7 μm.

(a) Use the principle of detailed balance to find the rate coefficient for collisional excitation as a function of temperature in degrees Kelvin. [Hint: dont forget to consider the degeneracy of the levels].

(b) Neglecting stimulated radiative excitation and de-excitation, and considering just these two levels in the carbon atom, derive an expression for n_u/n_l as a function of the density of neutral hydrogen atoms n_H and temperature T. Sketch n_u/n_l as a function of n_H for $T = 300$ K, 100 K and 30 K, and show that there is a critical density n_{cr} such that for $n_H \gg n_{cr}$, the CII levels approach LTE (local thermodynamic equilibrium: relative population of states given by the Boltzmann factor), while for $n_H \ll n_{cr}$, the upper level is severely depleted relative to the LTE population. Give n_{cr} in cm^{-3}.

(c) Asuming that the gas cloud is optically thin, compute the rate of cooling per unit volume by the [CII] 157.7 μm line as a function of n_H and T . By number, the interstellar abundance of carbon is

[1] 1the notation for spectroscopic terms is $^{2S+1}L_J$, so our transition is between states which both have $S = 1/2$ and $L = 1$, while $J = 3/2$ for the upper state and $J = 1/2$ for the lower state.

3×10^{-4} that of hydrogen. Assume that all carbon is singly ionized, and in one of the two ground state levels.

(d) Give the limiting forms of your general expression in (c) for $n \gg n_{cr}$ and $n \ll n_{cr}$, and explain physically why they have the forms that they do.

(e) In the outer parts of a dense photodissociation region in the Orion nebula, $n_H = 2 \times 10^5$ cm^{-3}, $T = 10^3$ K, and the rate of photoelectric heating is 3×10^{-17} erg cm^{-3} sec^{-1}. If these parts of the cloud are in thermal equilibrium (heating rate equals cooling rate, so temperature is not changing in time), what percentage of their total radiated luminosity is in the [CII] 157.7 μm line?

4. **Effects of non-zero optical depth.**

(a) Consider the same [CII] 157.7 μm transition as in the previous problem. In parts (c), (d), and (e) above, we assumed that all the line photons escaped. Show that if the cloud has a hydrogen column density[2] $N_H > N_c$, the optical depth at the center of the line will exceed unity, and the photons will no longer escape freely. Compute N_c assuming: $n_H \ll n_{cr}$, the line width is dominated by turbulent motions of velocity width Δv in km/sec, and that all the carbon is singly ionized. Give the limiting forms of your result for the cases i) $n_H \ll n_{cr}$ and ii) $n_H \gg n_{cr}$ when also $kT \gg h\nu_{ul}$. In case (i), give a numerical value in cm^{-2} for N_c.

(b) Use your result from (a) to compute the neutral-hydrogen column density at which the 21cm hyperfine-structure line of hydrogen will become optically thick ($\tau_\nu > 1$) at line center. Again assume that the line width Δv (in km/sec) is determined by turbulence. For this transition, $A_{ul} = 2.85 \times 10^{-15}$ sec^{-1}. Explain which of case (i) or case (ii) in (a) applies. Typical spin temperatures (see Bowers & Deeming pp. 364–365) for hydrogen clouds in the galaxy are $T_s \sim 10^2 - 10^3$ K. [Hint: the answer is $N_c = 1.8 \times 10^{18}\, T \Delta v$ cm^{-2}, where Δv is in km/sec and T in K.]

(c) What value does the intensity at line center have when the optical depth τ_ν at line center through the cloud is $\tau_\nu \gg 1$.

[2] Column density is the integrated number of atoms along a line of sight, say the z-axis: $N_H = R n_H dz$, and has units of cm^{-2}.