The Interstellar Medium

Problem Set 6

Due **in class** Wednesday February 16, 2011

Readings: Chapter 12.3 to 12.6 of Tielens.

Homework Problems:

Shock waves

1. It was stated in class that if x = ρ_1/ρ_2 is the density change across a shock front then x is the solution of a quadratic equation:

$$ax^2 + bx - c = 0$$

   a) Derive this equation.

   b) Show that the jump in pressure is:

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}$$

   c) Derive a similar equation for the jump in temperature T_2/T_1 in terms of γ and the Mach number M_1 of the shock.

2. A shock wave produced by a supernova explosion is moving into neutral atomic gas at $250\,\mathrm{km\,s^{-1}}$.

   a) Rewrite the jump condition for energy flux given in class to include the effect of ionization of hydrogen (denote the ionization potential by χ).

   b) Estimate the temperature immediately behind the shock, both (i) with, and (ii) without taking into account the ionization energy.

   c) Estimate how fast the shock must be moving in order to ionize hydrogen.

   d) For case (i) above, estimate the thickness of the layer in which the gas behind the shock cools from $10^6\,\mathrm{K}$ to $10^4\,\mathrm{K}$, assuming that the number density of the unshocked gas is $1\,\mathrm{cm^{-3}}$ and that the cooling rate as a function of temperature can be described by a single value (which you may choose from Figure 2.10 in Tielens). Remember that the density of the gas increases as it cools.

   e) Roughly at what range of wavelengths is most of the radiation emitted?

3. Sedov-Taylor phase of supernova remnants.

   a) Dimensional analysis gives the form of the Sedov-Taylor evolution, but leaves an unknown dimensionless number. This can be obtained from exact numerical solution of the hydrodynamic equations, as outlined in class, but one can also make a rough estimate as follows. Assume (inexactly) that the interior of the remnant is homogeneous. The total energy is then given by $E_{SN} = M(u_T + u_k)$, with M the total swept up mass, and u_T and u_k the thermal and kinetic energies per unit mass. Set these equal to the values just behind the shock front, $1.5P1/\rho_1$ and $v_1^2/2$. Then using the strong shock conditions, and recalling that the expansion velocity is equal to dRs/dt, you should recover equation 12.79 and a specific numerical estimate for ξ_0. (this estimate for ξ_0 is an overestimate, since the central pressure is actually lower than the postshock pressure).

   b) For the Sedov-Taylor phase of a supernova blast wave expanding into an intercloud medium $n = 0.5cm^{-3}$ at 1000 km s^{-1} , evaluate the cooling timescale and compare this to the expansion timescale.

   c) The Cygnus loop supernova remnant, shown in optical emission at http://zebu.uoregon.edu/~imamura/122/images/cygnus veil.jpg and in X-rays at http://www.iras.ucalgary.ca/~leahy/xraylist.html is a middle-aged supernovae remnant. The distance is uncertain, but the best estimate gives the remnant a diameter $\sim$ 10 pc, observed X-ray luminosity $L_X \sim 10^{36}$ erg/s and gas temperature $T_X \sim 3 \times 10^6$ K. As in part (a), ignore the interior density structure and derive an expression for the luminosity of the supernova remnant in terms of the cooling rate $\Lambda(T)$, density of the surrounding inter- stellar medium n_0 and the remnant radius R_s . Use this expression to determine the density of the medium surrounding the Cygnus loop, and the mass swept up by the supernova remnant.

4. Wind bubbles.

   During the adiabatic phase of a hot wind bubble, the expansion is governed by the energy and momentum equations (eqs 12.124 and 12.125 of Tielens). Assuming that the size of the remnant varies as t^η with constant η, show that $\eta = 3/5$ for a self-similar flow. Explain why this expression is similar to that describing the adiabatic phase of a supernova remnant (eq 12.79).