Astronomy 127

Spring 2010

Cosmology and Galaxy Formation

Final Exam

Due in Marc's office (Cahill 322) or mailbox (or to Shirley Hampton in 323 Cahill) by 12:00 noon on *Monday, June 7, 2010*

Please take 3 contiguous hours to complete the exam, at your convenience. You can use class notes and homeworks/solutions; please do not use any books. A calculator is OK. The first section is comprised of 7 problems from which you should work 3 (75 points), the second of 4 discussion questions of which you should answer 1 (25 points).

I. Problems (answer 3 out of 7) (75 points)

1. Suppose inflation is driven by a scalar field ϕ with potential $V(\phi) = g\exp(-\lambda\phi^2)$. What are the conditions on λ and ϕ to allow the use of the slow-roll approximation? Assuming these conditions to be satisfied, give values for the scalar and tensor slope parameters n_s and n_t. By how many e-foldings does the Universe expand when ϕ changes by a factor of 100?

2. In the 1990s, NASA's BATSE gamma-ray telescope observed no gamma-ray bursts that repeat within a timescale of a few hours to a few years after the first burst. However, if the dark matter in the Universe consisted of objects of some given mass M, we might have expected some of these bursts to have undergone gravitational lensing and thus seen "echos" of the original burst. Estimate what dark-matter mass range is ruled out from the non-observation of gamma-ray-burst repeaters.

3. Suppose somebody told you that the Universe is flat and the nonrelativistic-matter density is $\Omega_m \simeq 0.3$, but that the rest of the mass is in radiation, $\Omega_r \simeq 0.7$. What would be the angle subtended by the horizon at the surface of the CMB last scatter?

4. Suppose a QSO at redshift $z = 3.0$ has an observed flux of 10^{-26} ergs s^{-1} cm^{-2} Hz^{-1} at the rest-frame Lyman limit of hydrogen (912 Å; 13.6 eV; 1 Rydberg). It is observed that there is a marked deficit of Lyman-α–forest absorption lines within 5000 km sec^{-1} of the QSO emission redshift. Estimate the intensity of the "metagalactic" radiation field at the Lyman limit (in cgs units as above, per steradian) from this observation.

5. It is observed that there are approximately 2 galaxies per square arc minute which satisfy the color criteria for "Lyman break galaxies" in the redshift interval $2.7 \lesssim z \lesssim 3.4$, down to an observed flux of $\sim 2 \times 10^{-30}$ ergs sec^{-1} cm^{-2} Hz^{-1} (200 nJy) at rest-frame 1500 Å ($R_{\mathrm{AB}} \sim 25.5$ for the astronomers). Calculate the space density and approximate UV luminosity for a typical member of this population (in solar luminosities) for both an Einstein-de Sitter universe and an $\Omega_m = 0.3$ flat universe. Compare these numbers to the characteristic luminosity and space density of present-day galaxies, and comment on the differences between the two cosmological models considered.

6. Suppose that the total pressure and energy density of the Universe are related by $p = -\rho + \rho^2/\rho_1$, where ρ_1 is a constant. Assume zero spatial curvature. Calculate ρ as a function of the Robertson-Walker scale factor a, taking $a = a_1$ when $\rho = \infty$. Calculate a and ρ as functions of time, taking $t = 0$ as the time when $\rho = \infty$. Calculate the age of the universe and the deceleration parameter q_0 for a given present density ρ_0.

7. Estimate the fraction of the critical energy density in starlight in the Universe today. Make your assumptions and approximations clear. How does this compare with the energy density in the CMB?

8. Write an expression for the CMB temperature power spectrum C_l^{TT} in terms of the temperature autocorrelation function $C^{TT}()$, both for the full sky and in the flat-sky limit.

II. Discussion Questions – Answer 1 out of 3 (25 points)

1. Explain, using a combination of words, equations, and diagrams, how one would use observations of the Sunyaev-Zelodovich effect from clusters of galaxies as a means to measure the Hubble constant. What additional observations would be required? What are the sources of uncertainty?

2. Discuss the various pieces of evidence that point to the existence of non-baryonic cold dark matter as the dominant matter constituent of the universe (for example, what properties of CDM make it attractive, what are the problems that it solves, how does it do when confronted by observations of large scale structure, and what are the key pieces of evidence that tell us how much of it there must be?).

3. Discuss the various pieces of evidence and lines of reasoning for the existence of an accelerated cosmological expansion. Discuss the robustness of the various pieces of evidence, the pitfalls of some of the lines of reasoning, and then also discuss the possible explanations for cosmic acceleration.

4. Describe the principal features of the CMB temperature and polarization power spectra, where they come from, how they depend on the cosmological parameters, and how they can be used to learn about inflation (or anything else).