

Cosmology (Ay127), Spring 2010

Problem Set 1

Due: In class, first class of week 2

1. Show that the two forms of the Friedmann equation lead to identical dynamics. Do *not* assume that the matter is only pressureless.
2. Although general relativity is not required for this class, here's a simple exercise with metric that doesn't really require relativity. Find a coordinate transformation that shows that the Milne spacetime (i.e., the FRW spacetime with Friedmann equation $H^2 \propto a^{-2}$) is equivalent to a Minkowski spacetime. Explain what is going on.
3. The age t_0 of the Universe in the standard cosmological model depends on the current value of the Hubble parameter, $H_0 = 100 h$ km/sec/Mpc, as well as on Ω_0 , the current density in units of the critical density. In class, we showed that if $\Omega_m = 1$ and $\Omega_\Lambda = 0$, then $t_0(h) = 6.7 h^{-1}$ Gyr. (a) Your assignment is to generalize this result and derive an expression for the age of the Universe for $\Omega_m > 1$ and for $\Omega_m < 1$, both for $\Omega_\Lambda = 0$. (This shouldn't be too tricky—the answers are in the books. But still, you should derive the equations yourself.) Then, plot contours for $t_0 = 10$ Gyr, 13 Gyr, and 17 Gyr on the $\Omega_m - h$. You may do this either by sketching the contours by hand, or you may generate such a plot with Mathematica, C, Fortran, or anything else, if you're so inclined. (b) Then, make the analogous plots, but for $\Omega_m + \Omega_\Lambda = 1$ (and restricting to $\Omega_m < 1$). (c) Stellar astrophysicists believe that the oldest stars are around 10–20 Gyr. If the correct value is somewhere around 14 Gyr, your plots should show you for which values of Ω_m and h there might be consistency. (d) A few years ago, some astronomers found a galaxy at a redshift $z = 1.5$ with a spectrum well fit by stellar-population model with an age 3.5 Gyr. Draw on your plots the regions of the $\Omega_m - h$ parameter space ruled out by this measurement.