

Cosmology (Ay127), Spring 2010

Problem Set 2

Due: In class, first class of week 3

Suggested Reading: Ch 13 of Peebles' "Principles of Physical Cosmology". Ch. 8 of Longair, and Chs 2-3 of Kolb and Turner's "The Early Universe" are also good. Chs 1-2 of Dodelson's book.

1. The brightness of sources are measured on a logarithmic *apparent magnitude* scale, where the apparent magnitude is defined to be $m = -2.5 \log f + \text{constant}$, and f is the flux, and the logarithm is base 10. The luminosity of the source is similarly measured on a logarithmic *absolute luminosity* scale, where the absolute luminosity is $M = -2.5 \log L + \text{constant}$. The constants are chosen so that the *distance modulus* is $m - M = 5 \log(r/10 \text{ pc})$. If there is a *standard candle*, an object of known luminosity L (and thus known M), then measurement of its apparent magnitude m determines the luminosity distance. Suppose now that observers measure the distance modulus of supernovae (assumed to be standard candles) at redshifts $z = 0.5$ and $z = 1$. Calculate the luminosity distances at these two redshifts for (i) an Einstein-de Sitter Universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$), (ii) a flat Universe with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, and (iii) an open Universe with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0$. Determine the differences in the distance moduli between these three cosmologies, both at redshifts $z = 0.5$ and $z = 1$. Do some reading or talk with some of the local advanced grad students, professors, or postdocs to figure out how well the magnitude scale is calibrated and also to figure out what a typical extinction is and how accurately it can be subtracted from reddening measures.
2. In the standard cosmological scenario, all the electrons and protons in the Universe combine to form hydrogen at a redshift $z \simeq 1000$ in a process we call "recombination". However, at some redshift z_{reion} , stars begin to form and emit radiation that ionizes all the hydrogen in the Universe. If so, then cosmic microwave background (CMB) photons may Thomson scatter from the free electrons en route from the surface of last scatter. Calculate the optical depth $\tau(z_{\text{reion}})$ for Thomson scattering of CMB photons as a function of the reionization redshift z_{reion} for $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Derive an analytic approximation for $z_{\text{reion}} \gg \Omega_m^{-1}$ and make sure your exact expression agrees with this answer in the appropriate limit. Write your answer in terms of the baryon density $\Omega_b h^2 \simeq 0.023$ (where h is the Hubble parameter in units of 100 km/sec/Mpc) and in terms of the helium mass fraction $Y \simeq 0.23$. At what z_{reion} does $\tau = 1$?
3. Calculate the proper volume between redshifts $z = 3$ and $z = 3.1$ in

a square-degree field for (a) an Einstein-de Sitter Universe, (b) a flat Universe with $\Omega_m = 0.3$ and a cosmological-constant density $\Omega_\Lambda = 0.7$, (c) an open Universe with $\Omega_m = 0.3$, and (d) a Universe with a nonrelativistic-matter density $\Omega_m = 0.3$ and some other exotic matter with equation-of-state parameter $w = -1/3$ and density $\Omega_{w=-1/3} = 0.7$.

4. When a supernova goes off, it injects roughly 10^{51} ergs of kinetic energy into the surrounding interstellar medium (ISM), thus driving a shock wave into the ISM that then heats the ISM. The heated gas then cools primarily by Brehmsstrahlung emission. To a much lesser degree, the electrons in the gas can also cool by inverse-Compton scattering cosmic microwave background (CMB) radiation. Today, (at redshift $z = 0$) the CMB has a temperature $T = 2.7$ K. However, at earlier times, at redshift z , the energy density of the CMB will be much higher and the efficiency of inverse-Compton cooling of the supernova remnant will be much higher.

Estimate the redshift at which inverse-Compton cooling becomes the primary avenue for the supernova remnant to cool. If you do this sufficiently carefully, you could write an *ApJ* article about it; if so, you are spending too much time on this problem. We're just looking here for a back-of-the-envelope estimate, which should not be too difficult. Its more important here to get a well-reasoned rough answer than a very precise detailed answer—just be sure to make your assumptions and approximations clear. You may need to recall some results from your Radiative Processes and, if you're a bit more ambitious, ISM classes.