Cosmology Ay127, Spring 2010
Problem Set 6
Due: first class of week 7

1. In the CMB temperature power spectrum C_ℓ, there is a characteristic multipole moment ℓ (or equivalently, angular scale, $\theta \simeq \pi/\ell$) that separates small angular scales from large angular scales. That multipole moment is the $\ell_{\rm sh}$ corresponding to the sound horizon at the surface of last scatter at redshift $z \simeq 1100$. Usually when you see plots of the CMB power spectrum, it is $\ell^2 C_\ell$ that is plotted. For $\ell \lesssim \ell_{\rm sh}$, $\ell^2 C_\ell$ ∼constant, and for $\ell \gtrsim \ell_{\rm sh}$ there are a series of acoustic peaks that result from oscillations in the photon-baryon fluid before recombination. The multipole moment $\ell_{\rm sh}$ is, roughly speaking, the value of ℓ at which the first acoustic peak in the CMB power spectrum appears. (a) Calculate the angle $\theta_{\rm sh}$ $(= \pi/\ell_{\rm sh})$ subtended by the *sound* horizon at the surface of last scatter as a function of Ω_m and Ω_Λ. Assume that the sound speed in the baryon-photon fluid is $1/\sqrt{3}$ times the speed of light. (b) Plot iso-$\ell_{\rm sh}$ contours in the Ω_m-Ω_Λ parameter space. (c) Next, calculate the sound speed in the baryon-photon fluid just before recombination. How far does it differ from $1/\sqrt{3}$ for the currently preferred values for Ω_m and Ω_b? Which way would the first acoustic peak in the CMB power spectrum move if Ω_b was increased? (d) Suppose that instead of a cosmological constant (i.e., equation-of-state parameter $w = -1$), the dark energy had $w = -0.9$. Which way would the first acoustic peak move (holding all other parameters fixed)? You can answer this simply in words; no need for detailed calculation.

2. There are now several publicly available codes to calculate the CMB power spectrum C_ℓ; for example, two I know of are called CMBFAST and CAMB. Get a hold of any such CMB code by any means (e.g., you can google it, download, and compile it, or just get it from a friend). Then, run the code for currently favored values of the cosmological parameters and make plots of $\ell^2 C_\ell$. How does the power spectrum change if the primordial spectral index n for density perturbations is increased/decreased slightly from the Peebles-Harrison-Zeldovich value $n = 1$? What happens if the Hubble parameter is increased/decreased slightly? What happens if Ω_m is increased/decreased slightly (keeping the Universe flat)? What happens if the neutrinos get a small mass? What happens if $\Omega_b h^2$ is increased/decreased slightly? What happens if there are gravitational waves (tensor modes)? The point of this exercise is to simply play around with the numerical results to get a feel for how the power spectrum depends on cosmological parameters so you can better appreciate the meaning and implications of plots of measurements of the power spectrum when you see them in talks.