Cosmology Ay127, Spring 2010
Problem Set 7
Due: first class of week 8

1. Suppose that the cosmological dark matter consists of objects of mass M_d. Calculate the probability that a quasar at redshift $z = 3$ is gravitationally microlensed by one of these objects. Do this two ways: (a) first use a back-of-the-envelope calculation to show that this should be of order unity; (b) then do the calculation precisely using best-fit values for the cosmological parameters (and here, to be precise, "gravitationally microlensed" means that the source falls within the Einstein radius of the lens).

2. Suppose a star in the LMC is gravitationally microlensed by a dark halo object of mass M_d. (a) Calculate the dependence of the source brightness on time as the lens passes near the line of sight. (b) Derive an expression for the FWHM of this light curve in terms of the transverse velocity of the lens, the lens mass, and the lens distance. (c) What does the peak magnification amplitude depend on? (d) What is the separation of the two images in arcsecs? (e) Determine how the centroid of the light distribution changes during the microlensing event. (f) List at least four physical effects that might cause the actual time-dependence of the light curve to differ in detail from that calculated in part (a).

3. Consider weak gravitational lensing of some background population of galaxies at a redshift $z \sim 1$. In the weak-lensing regime, the amplification is $A(\vec{\theta}) = 1 + \epsilon(\vec{\theta})$ as a function of position $\vec{\theta}$ on the sky and with $\epsilon(\vec{\theta}) \ll 1$. If $\epsilon > 0$, then you'll be able to see fainter, and thus more, sources. However, if $\epsilon > 0$, then you are also magnifying the region behind the lens and thus seeing less volume for the same amount of area on the sky. Suppose that the source population has a luminosity function $(dN/dL) \propto L^{-\alpha}$. What is the condition on the value of α so that the sky density of background sources increases (decreases) for $\epsilon > 0$?

4. Suppose an effort is made to use weak gravitational lensing to construct a mass map of a galaxy cluster of mass $M = 5 \times 10^{14}\, M_\odot$ at a redshift $z = 0.2$. (a) Calculate the shear γ as a function of radius (in arcmins) from the cluster center (you may assume it is a point mass). (b) Suppose there are N imaged background sources (which you may assume to all be at redshift $z = 1$) per square arcmin on the sky (typical numbers are $10^4 - 10^5$ per square arcmin) and suppose furthermore that each of these sources has an intrinsic ellipticity $e = 0.1$. Calculate the smallest γ that could be detected at 3σ in a $\theta \times \theta$ sized patch of the sky (θ may here be something like $10''$ to a few arcmin).