Week 8: The Intergalactic Medium, Lyman-alpha forest, the first stars, and reionization

Cosmology, Ay127, Spring 2010

May 13, 2010

1 The Lyman-alpha forest

Quasars are extremely luminous cosmological sources powered by accretion onto supermassive black holes. Their extraordinary luminosities make them visible out to redshifts $z \sim 6$. Unlike stellar populations, which emit most of their energy in a relatively small range of frequencies, quasars emit over a very broad range of frequencies. Although they are quite interesting in their own right, quasars are useful tools for cosmology, as they serve as luminous beacons at huge distances which illuminate the nature of the intergalactic medium along the line of sight.

In particular, quasars typically have a very strong Lyman-alpha emission peak at a wavelength $\lambda = 1216$ angstroms. There is then emission at higher frequencies (or shorter wavelengths) that is, to a very rough first approximation, constant in intensity. The striking feature of the observed spectra, however, is a series of absorption lines at wavelengths longer than 1216 angstroms. These are interpreted to be due to Lyman-alpha absorption by neutral-hydrogen (HI) clouds at redshifts smaller than the quasar; i.e., absorption by hydrogen gas along the line of sight. Between these lines, the absorption is estimated to be $\tau \lesssim 0.1$ for redshifts $z \lesssim 4$. This is a very important observation, as it tells us that *almost all the hydrogen in the intergalactic medium is ionized at redshifts* $z \lesssim 5$. The argument, due to Gunn and Peterson, is pretty simple.

First, recall that the cross section for absorption of a photon by the Lyman-alpha resonance is (e.g., Peebles, Section 23) $\sigma = (3/4)\Lambda\lambda_\alpha^2\delta_D(\omega - \omega_\lambda)$, where ω is the (angular) frequency, $\Lambda = 6.25 \times 10^8\,\mathrm{sec}^{-1}$ is $2p \rightarrow 1s$ decay rate, and $\lambda_\alpha = 2\pi/\omega_c = 1216$ angstroms. Now consider a photon that initially has a wavelength $\lambda < 1216$ angstroms. The Universe then expands and the wavelength increases until it passes through the Lyman-alpha resonance. The optical depth for absorption as it passes through the resonance is $\tau = \int n_I(t)\,\sigma\,c\,dt$, where the integral is taken only over the time at which the photon passes through resonance, and $n_I(t)$ is the neutral-hydrogen density at that time. The time interval is $dt = da/\dot{a}$, and $\dot{a} = aH$, where $H \simeq \Omega_m H_0(1+z)^{3/2}$ is the expansion rate at $z \gtrsim 1$. The Dirac delta function in the expression for the cross section makes the integral easy, and the result is

$$\tau = \frac{3\Lambda\lambda_\alpha^3 n_I}{8\pi H_0 \Omega_m^{1/2}}(1+z)^{-3/2}, \tag{1}$$

or

$$n_I = 2.4 \times 10^{-11}\, \Omega_m^{1/2} h(1+z)^{3/2} \tau\, \mathrm{cm}^{-3}. \tag{2}$$

It is estimated from the continuum flux observed between the discrete Lyman-alpha absorbers that $\tau \lesssim 0.1$ at $z \lesssim 4$, from which we infer $n_I \lesssim 2 \times 10^{-12}\, \Omega_m^{1/2} h(1+z)^{3/2}\, \mathrm{cm}^{-3}$. Plugging in numbers, this implies that the fraction of hydrogen atoms that are neutral at redshifts $z \sim 5$ is only $\sim 10^{-7}$. If we include the hydrogen in the Lyman-alpha absorbers, this number increases a bit, but is still tiny compared with unity. Thus, at redshifts $z \lesssim 5$ (and possibly earlier), the hydrogen in the intergalactic medium is pretty close to neutral. In recent years, there have been several $z \gtrsim 6$ quasars discovered, and at $z \gtrsim 6$, the Gunn-Peterson trough ($\tau \gtrsim 1$) is beginning to appear. Care must be taken, however, that we do not immediately conclude that the Universe is neutral at $z \gtrsim 1$. Since hydrogen is such a good absorber, an optical depth $\tau \gtrsim 1$ implies only that the neutral fraction is $\gtrsim 10^{-6}$. The observations do seem to indicate that the increase in the neutral fraction is dramatic, so although we cannot conclude directly that the ionization fraction is much smaller at higher redshifts, the interpretation is that $z \sim 6$ is an era of rapid change in the neutral fraction at that the Universe at $z \gtrsim 6$ is neutral.

In retrospect, an ionized Universe at a redshift $z \sim 6$ should not be too surprising. As you showed in a previous homework, by a redshift of $z \sim 6$, the characteristic dark-matter mass in collapsed objects is $\sim 10^8\, M_\odot$, roughly a dwarf-galaxy. Such objects can produce stars quite efficiently. The important point is that once a star is formed, it is very efficient at ionizing its surroundings. Each baryon releases roughly 10 MeV as it is converted to iron in a star. This number can be increased if we take into account the gravitationally-driven radiation- and kinetic-energy input associated with each supernova. If this energy is released as ionizing photons ($E_\gamma \gtrsim 13.6$ eV), then there will be 10^6 ionizing photons released per baryon that is processed in a star. Of course, this estimate is degraded because (1) it is only OB stars which on the main sequence are hot enough to produce ionizing photons; (2) only ~ 0.1 of the mass in a star undergoes nuclear burning; (3) there is a finite efficiency for gas in halos to wind up in stars; and (4) since ionized atoms can recombine, (more detailed calculations suggest that) it actually takes ~ 10 ionizing photons to ionize an IGM hydrogen atom. Still, even if we ascribe a factor-of-ten reduction to the number of ionizations for each of these degradation factors, we still arrive at more than enough ionizing photons from star formation to ionize the Universe.

What ionizes the plasma? It is probably not collisional ionization, which is most efficient at temperatures $T \sim 10^6$ K and at high densities. As hinted above, it is probably the cosmic UV background. The precise origin of the UV background is still something that is debated. At redshifts $z \sim 1-2$, a significant fraction comes from quasars, but at higher redshifts, it is believed that a more significant fraction comes from stars. Either way, there is something called the *proximity effect* that is used to estimate the UV flux at high redshifts. The density of Lyman-alpha absorbers along the lines of sight to various quasars is consistent with a homogeneous distribution of clouds. The only exception is that near the quasar, the density of absorbers is seen to decrease. The interpretation is that the huge UV flux near the quasar photoionizes any clouds that are nearby. If that's the case, then we can conclude that at the distance from the quasar at which the absorber density decreases by 1/2, the cosmic UV-background intensity is equal to the UV intensity from the quasar. Near the ionization threshold ν_1, the mean cosmic ionizing flux is $i_\nu = i_{21}(\nu_1/\nu) \times 10^{-21}\, \mathrm{erg\, cm^{-2}\, sec^{-1}\, Hz^{-1}\, ster^{-1}}$, where the ν^{-1} spectrum approximates that from a quasar, and the dimensionless normalization is somewhere around $i_{21} \sim 1$ at redshift $z \sim 3$. This corresponds to a logarithmic number density per logarithmic bandwidth $n_\gamma = \nu n_\nu = 4\pi/(hc) = 6\times 10^{-5}\, i_{21}(\nu_1/\nu)\,\mathrm{cm}^{-3}$. Incidentally, recall that the baryon density is (for $\Omega_b h^2 = 0.05$) $n_b = 5.6\times 10^{-7}\,(1+z)^3\,\mathrm{cm}^{-3} = 3.6\times 10^{-5}\,\mathrm{cm}^{-3}$ at $z=3$; i.e., the number of ionizing photons is fairly similar to the number of baryons to be ionized.

Recalling that the cross section for photoionization of hydrogen is $\sigma_{\rm pi} = 7.9{\times}10^{-18}\,(\nu_1/\nu)^3\,\mathrm{cm}^{-2}$, the ionization rate per hydrogen atom is $\lambda_{\rm pi} = \int_{\nu_1}^{\infty}(4\pi i_\nu/h\nu)\sigma_{\rm pi}d\nu = 4\times 10^{-12}\, i_{21}\,\mathrm{sec}^{-1}$, corresponding to a mean life of $\sim 10^4\,\mathrm{yr} \ll H^{-1}$. While UV photons dissociate hydrogen atoms, electrons are also recombining with protons to form hydrogen atoms at a rate $\alpha n_p n_e$ per unit volume, where n_p and n_e are the proton and free-electron densities, and $\alpha = 4\times 10^{-13}\, T_4^{-0.7}\,\mathrm{cm}^3\,\mathrm{sec}^{-1}$ is the recombination coefficient and $T_4 = T/10^4\,\mathrm{K}$ where T is the plasma temperature. We set the mean hydrogen ionization rate $\lambda_{\rm pi}\langle n_I\rangle$ per unit volume equal to the mean recombination rate $\alpha\langle n_e n_p\rangle = C\langle n_p\rangle^2$, where C is a clumping factor (and $1/C$ is the filling fraction, the fraction of the total IGM filled by the gas) to find a neutral fraction,

$$\frac{\langle n_I\rangle}{\langle n_p\rangle} = \frac{\alpha C\langle n_p\rangle}{\lambda_{\rm pi}} = 1\times 10^{-6}\frac{C\Omega_{\rm IGM}h^2}{i_{21}T_4^{0.7}}(1+z)^3. \qquad (3)$$

Given the roughness of the approximations compared with the nastiness of the physical system, one should not read too much into this equation. Rather, it does seem to indicate that, to order of magnitude, the observed Gunn-Peterson limit to the neutral fraction is ballpark consistent with $\Omega_{\rm IGM}h^2 \sim 0.05$; i.e., with most of the baryons in this ionized IGM.

2 The absorbers

Damped Lyman-alpha systems—which show up as broad absorption troughs at the redshifted Lyman-alpha transition, redward of the quasar Ly-alpha emission line in the quasar spectrum—have neutral-hydrogen columns of $\Sigma \gtrsim 10^{20}\,\mathrm{cm}^{-2}$, comparable to the surface density of interstellar gas in a typical spiral galaxy. *Lyman-limit clouds* have column densities $\Sigma 3\times 10^{17}\,\mathrm{cm}^{-2}$. At these column densities, ionizing photons are absorbed longer than 912 angstroms, so these systems show up as a photoionization edge at the limit of the Lyman series of resonance lines. The most abundant absorbers are the *Lyman-alpha* clouds, which show up as a forest of narrow absorption lines at the redshifted Lyman-alpha frequency. These systems have column densities $\sim 10^{14}\,\mathrm{cm}^{-2}$.

The equivalent width W of a line is defined by $Wf_0 = \int(f_0 - f)d\lambda$, where f is the observed spectrum across the line, f_0 is the interpolated continuum spectrum at those wavelengths, and λ is the wavelength. If the optical depth at line center is $\tau \lesssim 1$ (either because the column density is low enough and/or because Doppler broadening makes the line shallower), then

$$W = \int \tau\, d\lambda = \Sigma\int \sigma d\lambda = \frac{3\Lambda}{8\pi}\frac{\lambda_\alpha^4}{c}\Sigma, \qquad (4)$$

in the rest frame of the absorber, which then gets increased by a factor $1+z$ when observed. E.g., for $\Sigma = 10^{13}\,\mathrm{cm}^{-2}$, $W = 0.05$ angstroms in the rest frame, and $W_0 = 0.2$ angstroms at redshift $z=3$.

For higher surface density, such that $\tau \gtrsim 1$, the exact expression is

$$W = \int d\lambda (1 - e^{-\tau}) = \int d\omega \frac{\lambda^2}{2\pi c} \left[1 - \exp\left(-\frac{3\lambda_\alpha^2 \Lambda^2 \Sigma}{8\pi[(\omega - \omega_\alpha)^2 + \Lambda^2/4]} \right) \right], \tag{5}$$

where we have used $\tau = \sigma\Sigma$ and the frequency dependence for the resonant Lyman-alpha scattering cross section. This expression evaluates to $W = 7.3(\Sigma/10^{20}\,\mathrm{cm}^{-2})^{1/2}$ angstroms. A column $\Sigma = 10^{21}\,\mathrm{cm}^2$ at $z = 2$ yields an observed EW of 70 angstroms. With a high-quality spectrum, the high-frequency "damping wings" ($\sigma \propto \omega^{-2}$) of the absorption trough can be fit.

The distribution of absorbers in column depth and redshift is approximated by $dP = g(\Sigma, z) d\Sigma dz$, where $g \simeq A\Sigma^{-\beta}$ (and Σ in units of cm^{-2}). At $z \sim 3$, $\beta \simeq 1.5$, and $A \simeq 10^{8.4}$ for $10^{13} \lesssim \Sigma \lesssim 10^{22}\,\mathrm{cm}^{-2}$. The slope of g steepens a bit above $\Sigma \sim 10^{20}\,\mathrm{cm}^{-2}$ and even moreso above $\Sigma \gtrsim 10^{22}\,\mathrm{cm}^{-2}$. If we had a fixed comoving density and physical-size distribution for absorbers, and an EdS universe, we would expect $g \propto (1+z)^3 (dt/dz) \propto (1+z)^{1/2}$. The observations suggest that $\Sigma \gtrsim 10^{20}$ systems behave not unlike this, while the evolution of the forest absorbers is much more rapid at low redshifts, implying that these more tenuous clouds are dissipating.

3 Damped Lyman-alpha systems

These things are most likely young Milky-Way–type galaxies. The total atomic-hydrogen column in these objects is

$$N_I = \int dz \int d\Sigma\, \Sigma g(\Sigma, z) = \int \frac{\langle n_I \rangle c}{H_0 \Omega_m^{1/2}} \frac{dz}{(1+z)^{5/2}}, \tag{6}$$

where the cosmic mean density of atoms in these clouds is

$$\frac{\langle n_I \rangle}{(1+z)^3} = \frac{A\Sigma_{\max}^{2-\beta}}{2-\beta} \frac{H_0 \Omega_m^{1/2}}{c(1+z)^{1/2}}, \tag{7}$$

assuming an upper cutoff $\Sigma_{\max}$ (note that with $\beta \simeq 1.5$, most of the hydrogen is in the largest absorbers). This density is $\Omega_I \sim 0.002(\Sigma_{\max,22}^{0.54})\Omega_m^{1/2}/h$ at $z \sim 3$, which is comparable to the density of luminous matter in galaxies today (note that column of protons through the Milky Way disk is $\sim 10^{22}\,\mathrm{cm}^{-2}$). The number of clouds with columns greater than Σ per unit redshift is $dN/dz = A\Sigma^{1-\beta}/(\beta - 1) \simeq 0.3\,\Sigma_{20}^{-0.46}$. If we assume a constant comoving density of L_* galaxies, $n = n_g(1+z)^3$ with $n_g = 0.01\,h^3\,\mathrm{Mpc}^{-3}$, and radii $10\,h^{-1}\,\mathrm{kpc}$, then

$$\frac{dN}{dz} = \frac{\pi r_g^2 n_g c}{H_0 \Omega_m^{1/2}} (1+z)^{1/2} \sim 0.02 \Omega_m^{-1/2}, \tag{8}$$

which is in reasonable agreement with the observations. Conclusion: these things are L_*-ish galaxies or proto-galaxies.

4 Lyman-alpha forest clouds

These are things that have hydrogen columns $\Sigma \sim 10^{14}\,\mathrm{cm}^{-2}$. Their z distribution is $dN/dz = B(1+z)^\gamma$ with $B = 3.5$ and $\gamma = 2.75$ for $\Sigma \gtrsim 10^{14}\,\mathrm{cm}^{-2}$ and $2 \lesssim z \lesssim 4$. This large value of γ

(cf., $\gamma = 1/2$ in EdS) shows that these clouds are dissipating over the observed redshift range. The mean proper distance between the clouds is

$$L = c\frac{dt}{dz}\frac{dz}{dN} \sim 0.6\,h^{-1}\Omega_m^{-1/2}\left(\frac{1+z}{4}\right)^{-5.25}\ \mathrm{Mpc}. \tag{9}$$

With sufficiently high-resolution spectra, the line-of-sight velocity dispersion in each absorber can be determined, and it is seen to be of order $b \sim 30$ km sec^{-1} corresponding to a gas temperature $T \sim 5 \times 10^4$ K, roughly what we would expect for photoionized gas clouds.

The density parameter for the neutral gas in $\Sigma \sim 10^{14}\,\mathrm{cm}^{-2}$ gas clouds is roughly $\Omega(n_I) \sim 10^{-7}\,\Omega_m^{1/2}/h$, comparable very roughly to the upper limit to the smoothly distributed neutral hydrogen, as we claimed before. However, these gas clouds are subjected to photoionizing radiation (and are optically thin to such radiation), and so there is probably a lot more ionized gas than the neutral gas we see. We will guess that the sizes of these clouds are $l = 10\,l_{10}h^{-1}$ kpc, where l_{10} parameterizes our ignorance and is probably in the range $1 \lesssim l_{10} \lesssim 10$. The lower bound is inferred by looking for coincident absorbers along neighboring lines of sight. The upper limit will be obtained below. In the homework, you will develop a better idea of the size of these clouds, and you will also find that most of the absorption takes place in the densest regions, so the clouds may be quite irregularly shaped, even though we assume them to be spherical.

We take as the canonical absorber $\Sigma = 10^{14}$ cm^{-2} and a redshift $z = 3$. The characteristic neutral-hydrogen density is $n_I \sim \Sigma/l \sim 3 \times 10^{-9}\,h/l_{10}$ cm^{-3}, and the characteristic neutral-hydrogen mass is $M_I \sim \Sigma l^2 m_p \sim 100\,l_{10}^2 h^{-2}\,M_\odot$; i.e., there is not that much neutral hydrogen here.

Let's now find the filling factor for these clouds. The mean density of neutral hydrogen in these clouds, averaged over all space, is $\langle n_I\rangle = \langle\Sigma\rangle\,(dN/dx)$, where $\langle\Sigma\rangle$ is the average column. But $dx = c\,dt$, and $dN/dx = (dN/dz)(dz/dx)$, and $dt/dz = \Omega_m^{-1/2}H_0^{-1}(1+z)^{-5/2}$. Then, the filling fraction is

$$\frac{1}{C} = \frac{\langle n_I\rangle}{n_I} = 10^{-5}\,(1+z)^{5.25} l_{10}\Omega_m^{1/2} = 0.02\,l_{10}\Omega_m^{1/2}, \tag{10}$$

the final equality being evaluated at $z = 3$. The number density of clouds is

$$n_{\rm clouds} = \frac{\langle n_I\rangle}{n_I l^3} = \frac{1}{Cl^3} = 2 \times 10^4\,\frac{h^3\Omega_m^{1/2}}{l_{10}^2}\,\mathrm{Mpc}^{-3}, \tag{11}$$

implying a characteristic cube size,

$$w_{\rm cl} = n_{\rm clouds}^{-1/3} = 40\,h^{-1}\Omega_m^{-1/6} l_{10}^{2/3}\,\mathrm{kpc} \tag{12}$$

at $z = 3$. We conclude that the clouds fill $\sim 2 - 10\%$ of space, implying that their spacing is not too much bigger than their sizes. Taking into account irregular shapes, it implies a cloudy IGM. Such a filling fraction also implies a characteristic mass overdensity $\delta_g \sim 10 - 50$. Therefore, if the gas traces the mass (and this is a *big* "if"), then these Lyman-alpha forest clouds correspond to nonlinear density perturbations, a bit past turnaround, but not yet virialized.

At 10^4 K, collisional ionization is ineffective, so the neutral fraction is

$$f \equiv \frac{n_I}{n_p} = \frac{\alpha n_p}{\lambda_{\rm pi}} = \frac{\alpha}{\lambda_{\rm pi}}\frac{\Sigma}{fl}, \tag{13}$$

giving us a neutral fraction,

$$f \sim \left(\frac{\alpha\Sigma}{\lambda_{\rm pi} l}\right)^{1/2} \sim \frac{2\times 10^{-5}}{T_4^{0.35}} \left(\frac{h}{i_{21} l_{10}}\right)^{1/2}. \tag{14}$$

The gas density in the cloud is

$$n_p \sim \frac{\Sigma}{lf} \sim 2\times 10^{-4}\, T_4^{0.35} \left(\frac{i_{21} h}{l_{10}}\right)^{1/2} \,\mathrm{cm}^{-3}, \tag{15}$$

and the characteristic gas mass is

$$M_{\rm II} \sim \frac{\Sigma l^2 m_p}{f} \sim 10^7\, T_4^{0.35} i_{21}^{1/2} \left(\frac{l_{10}}{h}\right)^{1/2} M_\odot. \tag{16}$$

Note that this is comparable to the stellar mass of a big globular cluster. The contribution to the cosmological density is

$$\Omega_{\rm II} \sim 0.005\, T_4^{0.35} \left(i_{21} l_{10} \Omega_m / h^3\right)^{1/2}, \tag{17}$$

comparable to the mass in high-column-depth clouds.

What keeps these clouds at their observed size? One possibility is pressure confinement by hotter IGM gas between the clouds (using the same physics as in the two-phase model for the ISM). This would require that the hot gas have a pressure $P/k \sim T n_p \sim 2\, T_4^{1.35} (i_{21} h / l_{10})^{1/2}$ K cm^{-3}. You will figure out in the homework why this is unlikely. Another, more likely, possibility is that the cloud is freely expanding. If it does so at velocity b (which is the velocity at which it would expand if it were not confined by gravity or pressure), then it would expand in a Hubble time by a fractional amount $(\delta l / l) \sim (bt/l) \sim (3/l_{10} \Omega_m^{1/2})$ at $z = 3$. This argument would not work for clouds at lower redshifts, when the expansion time is much longer. Such clouds may be gravitationally confined. This would require a total mass $M \sim kTl/(G m_p) = 2\times 10^8\, T_4 l_{10} h^{-1}\, M_\odot$, roughly ten times larger than the gas mass, and roughly, therefore, what one might expect for the associated dark-matter halo for these objects.

5 The warm-hot intergalactic medium (WHIM)

Note that the total amount of gas we infer in the Lyman-alpha forest is somewhat small compared with the mean baryonic density. This discrepancy becomes more severe at lower redshifts, when the Lyman-alpha forest disappears. This leads us to the "missing baryon" problem: i.e., where are all the baryons? (Incidentally, the contribution of stellar mass to the critical density is roughly an order of magnitude smaller than the CMB/BBN baryon density.) The most likely explanation is the *warm-hot intergalactic medium (WHIM)*. The characteristic dark-matter mass scale M_* is today something like $10^{13}\, M_\odot$, and the virial temperature for such halos is around $T \sim 10^{5.5-6}$ K. Gas at this temperature is ionized, and so does not appear in quasar absorption spectra. However, gas at this temperature emits in the far-UV or soft x-ray, frequencies which are difficult to detect, especially if the emission is diffuse. (Contrast this with x-ray emission from relatively dense $T \sim 10^7$ K gas in galaxy clusters, dark-matter halos of $M \sim 10^{14-15}\, M_\odot$, which appears very easily in x-ray telescopes.) The WHIM is thus very difficult to detect, and considerable efforts are now being made to detect it. Gas at these temperatures and densities is expected to have some highly ionized oxygen, which has absorption features at x-ray frequencies (Lyman-like transitions) and some at UV frequencies (fine-structure, I think). Just over the past few years, several groups have reported detections of absorption due to intergalactic OVI and OVII, and this has been interpreted as detection of the WHIM. However, these are very new results, with poor statistics, and it is difficult to draw precise quantitative conclusions from the data. Detection of these lines is difficult because they are weak and thus require large statistics. This requires that the background source be a very bright cosmological x-ray source for which the telescope integrates a long time, and the sources that are sufficiently bright are not too far.

6 The first stars

There is currently considerable interest in the first stars to form in the Universe and how they reionized the Universe. The problem is, however, theoretically extremely complicated, and observationally very poorly constrained at this point. There are therefore a very large number of interesting stories, but we cannot yet zero in on the "truth". In Ay123, we spent a bit of time the last week talking about stars formed from primordial gas, composed only of hydrogen and helium. We found that such stars are likely to be very massive and very hot, and to emit a nearly blackbody spectrum at temperatures $T \gtrsim 10^4$ K, producing many ionizing photons. These ionizing photons are then thought to be quite efficient at reionizing the Universe. Next week, we will (hopefully) see that primordial gas cannot cool on a timescale less than the Hubble time until the characteristic dark-matter–halo mass is $\sim 10^6\, M_\odot$. In a previous homework, you should have concluded that this characteristic mass M_* occurs at a redshift (very roughly) $z \sim 20$. If so, then these first stars would have formed and ionized the Universe. It is then believed (or argued), that formation of subsequent stars would then be shut off. The reason is that ionized gas cannot cool as effectively as atomic gas, and so one must wait until the characteristic mass scale is above $\sim 10^{8-9}\, M_\odot$ for the gas to cool quickly enough to form (what must be second-generation) stars.

The only observational handle we have currently on these epochs is the large-angle CMB polarization signal detected by WMAP. In an earlier homework, you found that the sound horizon at recombination subtends an angle $\theta \sim 1^\circ$, corresponding to a multipole moment $\ell \sim 200$, and that this result is obtained also by noting that the baryon-photon sound speed is at these times $\sim c/\sqrt{3}$. You also showed in an earlier homework that if all the gas in the Universe is reionized at a redshift of $z \sim 20$, and remains ionized, then the optical depth for CMB photon to Thomson scatter from reionized electrons is $\tau \sim 0.2$; i.e., that one in five (roughly) CMB photons re-scattered at $z \sim 20$ (recalling that most of the scattering happens, as you showed, at the higher redshifts). When these photons re-scatter at $z \sim 20$, the electrons from which they scatter will see a temperature quadrupole moment for the same reason that we do—i.e., from large-scale primordial density perturbations. And as you (should have) learned in Ay121, Thomson scattering induces a polarization in the scattered radiation that is proportional to (and roughly equal to one-tenth of) the quadrupole moment of the incident radiation. Therefore, the CMB photons scattered at $z \sim 20$ should have a polarization $\sim 10^{-5}/10 \sim 10^{-6}$, and since the fraction of photons scattered is $\tau \sim 0.2$, the observed polarization should be $\sim 2 \times 10^{-7}$. Moreover, the coherence scale of this polarization should be the coherence scale of the quadrupole moment at re-scattering, or the horizon at $z \sim 20$. Recalling that the angle subtended by the horizon at a redshift z is $\propto (1+z)^{-1/2}$, we infer that the characteristic multipole moment of the re-scattered polarization should be $\ell_{\rm reion} \sim 200\,(20/1100)^{1/2}/\sqrt{3} \sim 15$. Detailed calculations put this at slightly smaller ℓ, and when we take into account the fact that WMAP actually detects this large-angle polarization in cross-correlation with the temperature fluctuation (which is largest at small ℓ), the observed signal is at even slightly smaller ℓ. The bottom line, though, is that WMAP did in fact detect a large-angle CMB temperature-polarization cross correlation, and from the amplitude of their detection, we infer a reionization optical depth $\tau \simeq 0.17$ corresponding to a reionization redshift $z \sim 17$. Keep in mind, though, that their analysis is very tricky, the statistics marginal, and that the error bars are large. The data are still easily consistent with a reionization redshift as low as $z \sim 10$.

Ideas for probing the epochs $6 \lesssim z \lesssim 20$ (redshifts beyond the current Lyman-alpha forest, but below the expected first-star redshift) are abundant, although good ones are rare. Some ideas include looking at earlier galaxies with an imaging space telescope (i.e., JWST). Other ideas include studying IGM gas at these early epochs through absorption/emission features in the CMB due to the 21-cm transition in IGM hydrogen. The required frequencies would have to be quite low: $1.4\,(1+z)^{-1}$ GHz $\sim 50-200$ MHz. A number of projects (including the Square Kilometer Array) would be poised to detect these absorption features. People discuss looking simply for correlations (in both angular and redshift space) of the 21-cm features, and some people also talk of imaging, e.g., Stromgren spheres from the first ionizing sources.

7 The power spectrum from the Lyman-alpha forest

You may hear a lot these days about determination of the matter power spectrum $P(k)$ from the Lyman-alpha forest. This is a technique that has been applied to the huge quasar sample from the Sloan Digital Sky Survey (SDSS), although there were applications to smaller samples before that. The idea traces back to Croft et al. (ApJ, 1998). A combination of numerical simulations and semi-analytic reasoning leads us to believe that there is a relation between the low-density $\delta \sim 10$ gas that constitutes the Lyman-alpha forest and the underlying mass distribution (some of which you will investigate in the homework assignment). The observed Ly-alpha spectrum can thus be converted to a line-of-sight density field, with a normalization that is arbitrary. The three-dimensional power spectrum is then converted to a 3d power spectrum (as you worked out in a homework assignment). (3) Simulations are then run with the observed $P(k)$ with differing normalizations until the normalization matches the Ly-alpha power spectrum. The simulations/theory are constrained so that the mean Ly-alpha opacity matches the observations. The measurements need not be extremely high resolution. Roughly speaking, $\sim 1-10\,h^{-1}$ Mpc scales correspond to $100-1000$ km/sec, so measurements of the power spectrum can be done with spectral resolutions of 40 km/sec, well below state-of-the-art, and easily something achievable en masse with, e.g., SDSS.