Ph125c: Quantum Mechanics
Final Exam

Instructions: Please take no more than THREE (3) hours to complete the exam. You may consult with any of the class handouts, homeworks, or solution sets—please refer to nothing else. Please try to write your solutions as clearly as possible. Do the first three problems, and then choose one of problems 4 or 5. Please turn in your completed exam to me (Marc) in Bridge Annex 120. If I'm not there, you can leave it under my door, in my mailbox (outside Bridge Annex 151), or with Shirley Hampton in Bridge Annex 151. For seniors and graduate students, the exam is due by 5pm Friday, June 2. For ≤juniors, the exam is due by Friday, June 9.

1. Consider a three-dimensional attractive square-well potential, $V(r) = -V_0$ for $r < R$, and $V(r) = 0$ for $r > R$. Use the Born approximation to calculate the differential cross section $d\sigma/d\Omega$, as a function of scattering angle θ, for scattering of a particle of mass m and wavenumber k from this potential. Sketch your result for $d\sigma/d\Omega$ as a function of θ under the approximation that $kR \gg 1$.

2. Diagonalize the Hamiltonian $H = \omega a^\dagger a + \epsilon(ab^\dagger + ba^\dagger)$, where a and b are boson annihilation operators.

3. Consider electrons in the lowest Landau level of a quantum Hall sample in a magnetic field B. Suppose that pairs of electrons experience a repulsive (i.e., $V_0 > 0$) potential $V(r) = V_0$ for $r < R$ and $V(r) = 0$ for $r < R$. What is the largest quantum number m for the relative angular momentum between the two electrons for which $V(r)$ is nonzero? If the magnetic-field strength is $B = 10$ Tesla, how small does V_0 have to be (in eV) in order to neglect excitation to the next Landau level?

Choose **just one** of the following two questions:

4. Our Galactic halo consists of some mysterious dark matter with a mass density of about 0.3 GeV/c^2/cm^3. (GeV$= 1.6 \times 10^{-9}$ J.) The particles that make up this halo must be moving with a velocity dispersion of about 300 km/sec. One might guess that if one of the neutrinos has a nonzero mass, it could be the dark matter. Use the Pauli and Heisenberg principles to *estimate* the smallest allowable neutrino mass if the neutrino is to make up the dark matter in the Galactic halo. What do you think of this result?

5. A pencil is balanced precisely on its tip. Classically, this a static but unstable configuration. According to classical mechanics, if the pencil is not perturbed from this position, it can stay there forever. However, according to quantum mechanics (the Heisenberg uncertainty relation), the position of the pencil cannot be determined precisely, and so the pencil will eventually tip over. *Estimate* the longest time that the pencil can be balanced before it falls over. Does your answer make sense?