Ph125c

Spring 2006

QUANTUM MECHANICS

Problem Set 1

Due in class Wednesday, 5 April 2006

1. **The Lippman-Schwinger equation in 1 dimension:** The Lippmann-Schwinger formalism can be applied to a *one*-dimensional transmission-reflection problem with a finite-range potential, $V(x) \neq 0$ for $0 < |x| < a$ only.
   a. Suppose we have an incident wave coming from the left: $\langle x|\phi\rangle = e^{ikx}/\sqrt{2\pi}$. How must we handle the singular $1/(E - H_0)$ operator if we are to have a transmitted wave only for $x > a$ and a combination of the reflected wave and the original wave for $x < -a$? Is the $E \to E + i\epsilon$ prescription still correct? Obtain an expression for the appropriate Green's function and write an integral equation for $\langle x|\psi^{(+)}\rangle$.
   b. Consider the special case of an attractive δ-function potential,
   $$V = -\left(\frac{\gamma\hbar^2}{2m}\right)\delta(x), \qquad (\gamma > 0).$$
   Solve the integral equation to obtain the transmission and reflection amplitudes.
   c. What are the reflection and transmission amplitudes doing things the old-fashioned way (i.e., by solving the Schrodinger equation with the appropriate boundary conditions)? If you prefer to not solve this, you can find the solution from an old homework or book (e.g., Problem 3 from homework 3 in Ph125a, Exercise 5.2.3 in Shankar). Compare with the Lippmann-Schwinger result. Also, recall the value of the bound-state energy for this potential (for $\gamma > 0$).
   d. The one-dimensional δ-function potential with $\gamma > 0$ admits one (and only one) bound state for any value of γ, with a value determined in part (c). Show that the transmission and reflection amplitudes you computed have bound-state poles at the expected positions when k is regarded as a complex variable.

2. **Optical theorem and Born approximation:** Prove that the *total* cross section (i.e., the differential cross section integrated over angles),
$$\sigma_{\text{tot}} \simeq \frac{m^2}{\pi\hbar^4}\int d^3x \int d^3x'\, V(r)V(r')\frac{\sin^2 k|\mathbf{x}-\mathbf{x}'|}{k^2|\mathbf{x}-\mathbf{x}'|^2},$$
in each of the following ways:

a. By integrating the differential cross section obtained using the first-order Born approximation. (Hint: Keep in mind that the angle θ that you will be integrating the differential cross section over describes the direction of $\mathbf{k}'$, the wavevector of the scatter particle.)
b. By applying the optical theorem to the forward-scattering amplitude in the *second*-order Born approximation. [Note that $f(0)$ is real if the first-order Born approximation is used.]

3. **Born approximation for spherical charge distribution:** Use the Born approximation to express the differential cross section for scattering of an electron from a spherically symmetric charge distribution $\rho(r)$ as the product of the Rutherford cross section for a point charge and the square of a *form factor* F. Obtain an expression for (a) a uniform charge distribution of radius R, and (b) a Gaussian charge distribution with the same root-mean square radius.