Ph125c

Spring 2006

QUANTUM MECHANICS

Problem Set 2

Due in class Wednesday, 12 April 2006

This week's problems will focus on the method of partial waves.

1. **Square-well potential** (Sakurai 7.3): Consider a potential $V = 0$ for $r > R$ and $V = V_0$ =constant for $r < R$, where V_0 may be positive or negative. Using the method of partial waves, show that for $|V_0| \ll E = \hbar^2 k^2/2m$ and $kR \ll 1$ the differential cross section is isotropic and that the total cross section is given by

$$\sigma_{\rm tot} = \left(\frac{16\pi}{9}\right)\frac{m^2 V_0^2 R^6}{\hbar^4}.$$

Suppose the energy is raised slightly. Show that that angular distribution can then be written as

$$\frac{d\sigma}{d\Omega} = A + B\cos\theta,$$

and obtain an approximate expression for B/A.

2. **Partial-wave scattering from a Yukawa potential** (Sakurai 7.4): A spinless particle is scattered by a weak Yukawa potential,

$$V = \frac{V_0 e^{-\mu r}}{\mu r},$$

where $\mu > 0$ but V_0 can be positive or negative. The first-order Born amplitude is given by

$$f^{(1)}(\theta) = -\frac{2mV_0}{\hbar^2\mu}\frac{1}{[2k^2(1-\cos\theta)+\mu^2]}.$$

   a. Using $f^{(1)}(\theta)$ and assuming $|\delta_l| \ll 1$, obtain an expression for δ_l in terms of the Legendre function of the second kind,

$$Q_l(\zeta) = \frac{1}{2}\int_{-1}^{1}\frac{P_l(\zeta')}{\zeta-\zeta'}d\zeta'.$$

b. Use the expansion formula,

$$\begin{aligned}Q_l(\zeta) = &\frac{l!}{1\cdot 3\cdot 5\cdots(2l+1)}\\ &\times\left\{\frac{1}{\zeta^{l+1}} + \frac{(l+1)(l+2)}{2(2l+3)}\frac{1}{\zeta^{l+3}}\right. \\ &\left.+\frac{(l+1)(l+2)(l+3)(l+4)}{2\cdot 4\cdot(2l+3)(2l+5)}\frac{1}{\zeta^{l+5}} + \cdots\right\}, \qquad (|\zeta| > 1)\end{aligned}$$

to prove each of the following two assertions: (i) The phase shift δ_l is negative (positive) when the potential is repulsive (attractive). (ii) When the de Broglie wavelength is much longer than the range of the potential, δ_l is proportional to k^{2l+1}. Find the proportionality constant.

3. **Scattering by an impenetrable sphere** (Sakurai 7.6): Consider the scattering of a particle by an impenetrable sphere: $V(r) = 0$ for $r > a$ and $V(r) = \infty$ for $r < a$.
   a. Derive an expression for the s-wave ($l = 0$) phase shift. (You need not know the detailed properties of the spherical Besel functions to be able to do this simple problem.)
   b. What is the total cross section σ_{tot} in the extreme low-energy limit, $k \to 0$. Compare your answer with the geometric cross section πa^2.