Ph125c

Spring 2006

QUANTUM MECHANICS

Problem Set 3

Due in class Wednesday, 19 April 2006

This week's problems will contine with partial waves and also deal with resonant scattering and electron-atom scattering.

1. Consider a "shell potential,"

$$V(r) = \alpha\delta(r - r_0),$$

for r, the radial spherical coordinate in three dimensions.

a. Find the s-state wavefunction for $E > 0$. Include an expression that determines the phase shift δ_0. With $\hbar k = \sqrt{2mE}$, show that in the limit $k \to 0$, $\delta_0 \to ak$, with constant a (the "scattering length"). Solve for a in terms of α and r_0.

b. How many bound states can exist for $\ell = 0$ and how does their existence depend on α?

c. What is the scattering length a when a bound state appears at $E = 0$? What happens to a as the shell potential changes from repulsive ($\alpha > 0$) to attractive ($\alpha < 0$), and when α is sufficiently negative to form a bound state? Sketch a as a function of α.

2. In this problem you will consider the deuteron and neutron-proton scattering more carefully. The deuteron, a spin-triplet and $\ell = 0$ bound state of a neutron and a proton, has a binding energy of 2.26 MeV. It is also known that this is the only bound state of the neutron and proton (there are no excited bound states.) The neutron-proton scattering length (for the triplet) is $a_t = 5.42$ fm. Suppose we guess that the neutron-proton interaction is an attractive rectangular well of depth V_0 and radius R (i.e., $V(r) = -V_0$ for $r < R$ and $V(r) = 0$ for $r > R$.)

a. Find values for V_0 and R that reproduce this binding energy and scattering length.

b. Calculate the effective range r_0 you would expect from these parameters. How does this result compare with the experimental value of $r_t = 1.73$ fm?

c. What is the total cross section at low energies? and how does it compare with πR^2.

d. At what energies is it safe to approximate the total cross section just by the $\ell = 0$ cross section?

e. What is the total cross section for this potential at high energies? and how does it compare with πR^2?

3.

a. Derive equation (7.12.6) from Sakurai,

$$\frac{d\sigma}{d\Omega}(0 \to n) = \frac{1}{(\hbar k/mL^3)} \frac{2\pi}{\hbar} |\langle \mathbf{k}' n|V|\mathbf{k}0\rangle|^2 \left(\frac{L}{2\pi}\right)^3 \left(\frac{k'm}{\hbar^2}\right)$$
$$= \left(\frac{k'}{k}\right) L^6 \left| \frac{1}{4\pi} \frac{2m}{\hbar^2} \langle \mathbf{k}', n|V|\mathbf{k}, 0\rangle \right|^2,$$

the differential cross section for inelastic electron-atom scattering. Here, $\mathbf{k}$ is the wavevector of the incident particle (of mass m) and $\mathbf{k}'$ that for the scattered particle. The operator V describes the interaction potential between the incident electron and the atomic nucleus and electrons, and L is the size the (very large) box that is assumed to enclose the system.

b. (Sakurai 7.11) Show that the differential cross section for the elastic scattering of a fast electron by the ground state of the hydrogen atom is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{4m^2e^4}{\hbar^4 q^4}\right) \left\{1 - \frac{16}{[4 + (qa_0)^2]^2}\right\}^2.$$

Ignore the effect of identity of the incident and atom electron.