QUANTUM MECHANICS

Problem Set 4

Due in class Wednesday, 26 April 2006

Reading assignment: The *Scientific American* article by Bert Halperin ["The Quantized Hall Effect," B. I. Halperin, *Sci. Am.* **254**, 52 (1986)] that was handed out in class (and is available in Lauritsen 264). And also read Sections 1.1–1.5 in "The Quantum Hall Effect: Novel Excitations and Broken Symmetries," S. M. Girvin, `http://www.arxiv.org/abs/cond-mat/9907002` (also handed out in class and available in Lauritsen 264).

1. **Gaussian wave packet in a Hall sample:** (Girvin's exercise 1.5) Consider an electron moving in the x-y plane under the influence of a magnetic field $\vec{B} = B\hat{\mathbf{z}}$. Using the Landau gauge, construct a gaussian wave packet in the lowest Landau level of the form,

$$\Psi(x,y) = \int_{-\infty}^{\infty} dk\, a_k\, e^{iky} \exp\left[-\frac{1}{2\ell^2}(x + k\ell^2)^2\right],$$

   choosing a_k in such a way that the wave packet is localized as closely as possible around some point $\vec{R}$. Here, $\ell^2 \equiv \hbar c/eB$. What is the smallest size wave packet that can be constructed without mixing in higher Landau levels? If you've done things correctly, your wave packet should preserve the symmetry of the problem, even if it is broken explicitly by the choice of the vector potential.

2. **The integer quantum Hall effect:** (Girvin's exercise 1.7. It will help you to understand what is going on if you first read carefully Halperin's *Scientific American* article as well as Section 1.4 in Girvin's lecture notes. This is not necessarily an easy problem, but you should give it a shot; you'll have a healthy sense of accomplishment if you figure it out.) Suppose we had a two-dimensional quantum-Hall sample that had *no* impurities. Show that in this case, the number of edge channels whose energies lie in the gap between the two Landau levels scales with the length L of the sample, while the number of bulk states scales with the area. Use these facts to show that the range of magnetic field in which the chemical potential lies in between two Landau levels scales to zero in the thermodynamic limit (i.e., when the number of particles becomes huge). Hence finite-width quantum-Hall plateaus cannot occur in the absence of disorder that

produces a reservoir of localized states in the bulk whose number is proportional to the area.

3. **Non-commutative geometry:** (Girvin's exercise 1.8) Consider the motion of an electron in the x-y plane moving in a uniform magnetic field $\vec{B} = B\hat{z}$. In addition, there is a complicated non-uniform electric field in the x-y plane given by the gradient of some potential $V(x,y)$. Consider the limit in which the mass is taken to zero (so that the cyclotron frequency becomes large).

   a. Derive the classical equations of motion from the Lagrangian and show that they reduce to a simple $\vec{E} \times \vec{B}$ drift along isopotential contours.

   b. Find the momentum conjugate to the coordinate x and show that (with an appropriate gauge choice) it is the coordinate y:

$$p_x = -\frac{\hbar}{\ell^2} y,$$

   so that we have the strange commutation relation

$$[x, y] = -i\ell^2,$$

   where $\ell^2 \equiv \hbar c/(eB)$. In the infinite-field limit, where $\ell \to 0$, the coordinates commute and we recover the semi-classical result in which effectively point particles drift along isopotentials. (Note: This problem works out an elementary example of "non-commutative geometry," an idea that appears in condensed-matter theory and also amuses string theorists.)