Ph125c

Spring 2006

QUANTUM MECHANICS

Problem Set 5

Due in class Wednesday, 3 May 2006

Reading assignment: Section 1.6 in "The Quantum Hall Effect: Novel Excitations and Broken Symmetries," S. M. Girvin, `http://www.arxiv.org/abs/cond-mat/9907002` (handed out in class and available in Lauritsen 264).

1. **Haldane pseudopotential and Landau energy gaps** (Girvin's Exercise 1.12): Evaluate the Haldane pseudopotentials for the Coulomb potential $e^2/\epsilon r$. Express your answer in units of $e^2/\epsilon\ell$. For the specific case of $\epsilon = 10$ and $B = 10$ T, express your answer in Kelvin (using Boltzmann's constant to convert an energy to a temperature). Compare the value of the Haldane pseudopotential to the energy gap $\hbar\omega$ (where ω is the Larmour frequency) for $B = 10$ T. What do you make of your numerical results?

2. **Finite thickness of quantum Hall sample** (Girvin's exercise 1.13): Take into account the finite thickness of the quantum well by assuming that the one-particle basis states have the form,

$$\psi_m(z, s) = \varphi_m(z)\Phi(s),$$

   where s is the coordinate in the direction normal to the quantum well. Write down (but do not evaluate) the formal expression for the Haldane pseudopotentials in this case. Describe *qualitatively* the effect of finite thickness on the values of the different pseudopotentials for the case where the well thickness is approximately equal to the magnetic length.

3. **Three-dimensional correlation function for identical fermions:** This problem generalizes the exercise in class where we calculated the correlation function for spinless identical fermions in a one-dimensional box. Consider a gas of N (where N is a huge number) electrons in a cubical box of volume $V = L^3$. First assume that the electrons are all in the same spin state. Calculate the two-point correlation function $g(r)$ for the electrons; be sure your answer makes sense. Next, suppose that there is no magnetic field so that both spin-up and spin-down states are occupied. What is the two-point correlation function now?

4. **Correlation function for the $m = 1$ Laughlin state:** By writing the $m = 1$ Laughlin state as a single Slater determinant and integrating out all but two of the coordinates, show that the radial correlation function is [Girvin's equation (1.112)]

$$g(z) = 1 - e^{-|z|^2/2}.$$

Explain in a few words what this result means.