

**Ph125c**  
**Spring 2006**

**QUANTUM MECHANICS**

**Problem Set 6**

Due in class Wednesday, 10 May 2006

Reading assignment: Chapter 21 in Merzbacher's quantum mechanics book (handed out in class and available in Lauritsen 264). This week's problem set deals with formalism, and may not be the most exciting homework assignment, but starting next week we'll get to some pretty cool applications.

1. Fermionic creation and annihilation operators  $a^\dagger$  and  $a$  satisfy  $a^\dagger a + a a^\dagger = 1$ . Find the eigenvalues of the number operator  $N = a^\dagger a$ .
2. (Merzbacher Exercise 21.7) Show that the square of an additive one-particle operator is generally expressible as the sum of an additive two-particle operator and an additive one-particle operator.
3. (Merzbacher Exercise 21.8) Prove from the commutation relations that

$$\langle 0 | a_i a_j a_k^\dagger a_\ell^\dagger | 0 \rangle = \delta_{jk} \delta_{i\ell} \pm \delta_{ik} \delta_{j\ell},$$

the sign depending on the statistics, Bose-Einstein (+) or Fermi-Dirac (-). Also, calculate the vacuum expectation value,

$$\langle 0 | a_h a_i a_j a_k^\dagger a_\ell^\dagger a_m^\dagger | 0 \rangle.$$

4. (second half of Merzbacher Exercise 21.9, neglecting spin degrees of freedom and working in one, rather than three, spatial dimensions) Show that if the two-particle interaction operator  $V$  is *local* in coordinate space,

$$\langle r', r'' | V | r''', r'''' \rangle = V(r', r'') \delta(r' - r''') \delta(r'' - r'''),$$

then the expectation value of the additive two-particle operator  $\mathcal{V}$  in an  $n$ -particle state is

$$\langle \Psi^{(n)} | \mathcal{V} | \Psi^{(n)} \rangle = \sum_{i>j=1}^n \int |\psi(r_1, \dots, r_n)|^2 V(r_i, r_j) dr_1 \dots dr_n.$$